Uncertainty Shocks, Financial Frictions and Business Cycle Asymmetries Across Countries †

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Abstract

In this paper, I explore the interaction of uncertainty shocks and financial frictions in generating the excess volatility characterizing business cycle fluctuations in emerging countries vis-à-vis advanced countries. I use a small open economy model with nominal rigidities augmented with the financial accelerator mechanism. The model can generate the simultaneous decline in key macroeconomic variables in response to an uncertainty shock. The financial accelerator mechanism captures the asymmetry in borrowing costs experienced by emerging countries in global capital markets. This channel allows the model to generate the amplified response of real variables to uncertainty shocks for emerging countries in comparison to advanced countries. Furthermore, using Mexico and the United Kingdom as representative emerging and advanced countries, I estimate the strength of the financial frictions channel along with key behavioral parameters in generating the amplified responses of key macro variables to uncertainty shocks in recessions.

JEL Classification Codes: C32, E32, F41, E37

Keywords: Uncertainty Shocks, Financial Frictions, Emerging Countries, Recessions, Business Cycles.

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1 Introduction

The aftermath of the Great Recession has sparked an interest in exploring the role of economic uncertainty in generating business cycle fluctuations.¹ Policymakers in various speeches have suggested heightened economic uncertainty as the chief impediment in global recovery.² This recent strand of literature has established three main stylized facts that characterize the impact of uncertainty on the macroeconomy. First, an increase in uncertainty triggers a ‘wait and see’ response among agents leading to a simultaneous decline in consumption, investment and output. Second, emerging and low-income countries are more vulnerable to uncertain environments. Third, the effects of higher uncertainty matter more during downturns in the business cycle.

In addition to establishing the empirical relevance of uncertainty shocks, the literature has attempted to reconcile the consequences of uncertainty shocks within the framework of microfounded models. However, the emphasis has largely been focused towards generating the first stylized fact within the framework of closed economy models calibrated to match characteristics of developed countries such as the United States (Basu and Bundick 2017). In the context of international macroeconomics, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011) examine the role of interest rate uncertainty within the framework of a one sector real business cycle model.

The motivation of this paper is to primarily reconcile the asymmetry in the response of variables to upward surges in uncertainty between advanced and emerging countries. To reconcile the second stylized fact within a theoretical model I use a small open economy DSGE model with nominal rigidities and financial frictions. Financial frictions in this set-up interacts with uncertainty to generate the excess volatility of real variables observed in emerging countries vis-à-vis advanced countries. Furthermore, using the impulse response function matching technique, I quantify the role of financial market imperfections along with key behavioral parameters in the generating asymmetric effects of uncertainty shocks across advanced and emerging countries during recessions. By using Mexico and the United Kingdom as representative emerging and advanced countries respectively, I estimate the parameters of interest by matching model implied impulse responses to uncertainty shocks to impulse responses generated from the recessionary

¹Bloom 2009
²Christine Lagarde 2012, Richard W. Fisher 2013
regime of a Smooth Transition Vector Auto Regression Model.

In the theoretical specification, I introduce financial frictions by modeling borrowing costs as a function of a global component and a country-specific component as in Neumeyer and Perri (2005). I use the financial accelerator mechanism described in Gertler, Gilchrist and Natalucci (2007) to make the country specific component more responsive to the level of credit for emerging countries. The uncertainty in this model stems from the time-varying volatility of exogenous processes (preferences and aggregate productivity). An uncertainty shock in the model can therefore be interpreted as uncertainty about future preferences and aggregate productivity. The model is solved using perturbation methods, in particular, a third order Taylor Series expansion as suggested in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2016).

The main results that I present in this paper are threefold. First, the model can generate the key stylized fact about uncertainty shocks in a small open economy set-up with higher uncertainty leading to a simultaneous decline in consumption, investment and GDP. Second, I find that by varying the strength of the financial accelerator mechanism, the model can generate the amplified responses of real variables (consumption, investment and GDP) with strongly countercyclical trade balances that is characteristic of business cycles in emerging countries. My findings therefore emphasize the interaction of uncertainty shocks and financial frictions in generating business cycle asymmetries between advanced and emerging countries. Third, the results of the estimation suggest that amplified financial frictions as well as higher average uncertainty is characterize recessions for emerging and advanced countries alike. While estimates of average uncertainty are comparable for both Mexico and the United Kingdom, financial frictions are higher in magnitude for Mexico. These results emphasize the role of country fundamentals versus exogenously different shocks towards characterizing the excess volatility of real variables in emerging countries.

2 Existing Literature

The idea of uncertainty shocks as a driver of business cycle fluctuations has gained traction since the Great Recession. The seminal contribution by Bloom (2009) proposes the role of second moment shocks as a channel that generates a ‘wait and see’ response in real variables. In his framework, an increase in uncertainty triggers a sharp decline in real activity with medium term
volatility overshoot. Bloom (2014) provides several empirical proxies that capture the effects of macro as well as micro uncertainty. Furthermore, his findings suggest that uncertainty shocks are more important during downturns in the business cycle. This evidence is also presented by Jurado, Ludwigson and Ng (2015) where the authors construct a measure of macroeconomic uncertainty using the unforeseeable component of a large number of economic indicators. They demonstrate that their measure of uncertainty explains a much larger component of total uncertainty during recessions and exhibits stronger linkages with macroeconomic variables during recessionary episodes in business cycles.

While these studies provide empirical support and lay the theoretical foundation to demonstrate the role of uncertainty shocks in driving the dynamics of macroeconomic variables, Basu and Bundick (2017) explore the role of demand uncertainty in a one sector closed economy dynamic general equilibrium model. They demonstrate that in the absence of nominal rigidities, the model fails to generate the simultaneous decline in consumption investment and output which is characteristic of uncertainty shocks. They highlight that when output is demand determined, an increase in demand uncertainty spurs a precautionary savings motive among households that encourages savings and increase in labor supply. With other factors of production remaining fixed this increase must lead to an increase in output and investment in a closed economy model. With nominal rigidities they are able to break this divergence as firms respond by increasing markups in the face of demand uncertainty and subsequently leading to a decline in employment and hence output and investment. Bonciani and Roye (2016) use this intuition of sticky prices in a closed-economy general equilibrium model with a banking sector. They highlight that credit frictions amplify the effect of an uncertainty shock to preferences while generating the simultaneous decline in consumption, investment, and output. Born and Pfeifer (2017) highlight the precautionary pricing motive characterizing supply side uncertainty and demonstrate that in the presence of sufficient nominal rigidities in prices as well as wages, a model with supply side uncertainty leads to a decline in output.

These studies provide empirical and theoretical frameworks to analyze the role of uncertainty shocks on macroeconomic variables for closed economy models. In the context of international macroeconomics and business cycle asymmetries across advanced and emerging countries, the role of uncertainty shocks has been investigated to lesser extent.
The literature examining asymmetries in business cycles across advanced and emerging countries has evolved along two different sets of complementary approaches. On one hand the work of Aguiar and Gopinath (2007) emphasizes the differences in the nature of exogenous processes as the guiding factor in the observed asymmetry. The authors show that shocks to the trend of the productivity process is the main driver of business cycle fluctuations in emerging countries as opposed to advanced countries which are characterized by shocks to productivity that are stable about the trend. The other approach emphasizes that while underlying exogenous processes driving business cycles are similar across countries, differences in fundamentals such as weaker institutions, political instability, and unstable policy amplify the effect of a shock and drive the observed asymmetry between the two sets of countries. In this paper, I follow the latter approach in modeling differences the differences between the two types of countries.

In particular, financial frictions in different forms have been documented as an important channel of amplification. Neumeyer and Perri (2005) model the higher borrowing costs faced by emerging countries as a function of country-specific risks. Uribe and Yue (2006) underscore that the feedback from emerging country fundamentals to country spreads significantly exacerbate business cycle fluctuations. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011) build upon the results from Uribe and Yue (2006) and explore the uncertainty about interest rates through a stochastic volatility representation for Argentina, Brazil, Ecuador and Venezuela. The authors explore two dimensions of interest rate volatility: the volatility of the international risk-free rate and the volatility of the country-specific component of borrowing costs to find that the second channel is more relevant for the group of emerging countries examined. Furthermore, using a simple one sector RBC model, the authors demonstrate that upward surges in the volatility of interest rates lead to a fall in output, consumption, investment, hours worked and a notable change in the current account.

Another paper exploring this financial friction driven asymmetry in business cycles across advanced and emerging countries is that of Ordonez (2010), who uses a model of information friction with learning. Weaker financial conditions exacerbate recessions by amplifying information frictions. In the literature examining uncertainty shocks, Swallow and Cespedes (2013) examine the impact of uncertainty shocks within a SVAR framework. The main results suggest that uncertainty shocks lead to a larger and more persistent decline in real variables on average
for emerging countries vis-à-vis advanced countries.

The primary contribution of this paper is to examine the interaction of uncertainty shocks and financial frictions in generating business cycle asymmetries across advanced and emerging countries. In addition to generating the key stylized facts characterizing the impact of uncertainty for a small open economy model, I also quantify the strength of the two channels – financial frictions and alleviated levels of uncertainty along with estimating the key behavioral parameters that explain why recessions are deeper and more persistent for emerging countries vis-à-vis advanced countries.

The theoretical framework comprising a benchmark small open economy model augmented with a financial accelerator mechanism is closest to the set-up adopted in Gertler, Gilchrist and Natalucci (2007). I describe the model in detail in section 3. In section 4, I demonstrate the ability of the model to replicate the first two stylized facts about uncertainty shocks. First, an upward surge in uncertainty triggers a simultaneous decline in consumption, investment and GDP in a small open economy model. Second, financial frictions and uncertainty shocks interact to generate the asymmetric effect of uncertainty shocks across model calibrations corresponding to representative advanced and emerging countries respectively. In section 5, I match impulse responses generated from the model with impulse responses to uncertainty shocks from the recessionary regime of Smooth Transition Vector Auto Regression model to estimate the key parameters of interest guiding the asymmetry in the behavior of macroeconomic variables across the two types of countries in recessions.

3 Model Specification

This is a model in discrete time where agents live infinitely. There are four agents in this model economy - households, entrepreneurs, producers of capital goods and retailers. Households consume, supply labor and save in foreign and domestic assets. Entrepreneurs borrow from global credit markets and use a combination of net worth and foreign currency denominated debt to raise capital required for the production of wholesale goods. Capital producers purchase undepreciated capital at the end of each period from entrepreneurs, combine them with investment to meet the final capital demand from entrepreneurs. Retailers of domestically produced goods operate within a monopolistically competitive environment. They purchase
wholesale goods from entrepreneurs, costlessly differentiate them and sell the final composite
good to households, capital producers and rest of the world as exports. Retailers of imported
goods also operate within a monopolistically competitive environment and purchase wholesale
goods from rest of the world to costlessly differentiate and sell the final imported good to house-
holds and capital producers. I assume that the main difference between advanced and emerging
countries lies in the cost of credit faced in international capital markets and is specified in the
characterization of the entrepreneurial sector. The behavior of each type of agent is described
in detail as follows:

3.1 Households

Households maximize:

\[ U_t = E_0 \sum_{t=0}^{\infty} \beta^t z_t \left( \frac{(C_t - H_t)^{1-\rho}}{1-\rho} - \frac{L_t^{1+\psi}}{1+\psi} \right) \]

here, \( H_t \) denotes the level of habits.\(^3\) \( L_t \) denotes hours worked. I assume that habits are
external and evolve as function of aggregate consumption in the past, that is, \( H_t = hC_{t-1} \).
\( C_t \) is the consumption aggregate across domestic goods \( C_{H,t} \) and foreign goods \( C_{F,t} \). \( \frac{1}{\rho} \) is the
intertemporal elasticity of substitution for habit-adjusted consumption across periods. \( \beta \in (0,1) \)
is the discount factor and \( z_t \) is the shock to preferences. This can be interpreted to generate
demand specific uncertainty in the model.

There is a unit continuum of differentiated domestic goods and a unit continuum of differen-
tiated foreign goods such that the aggregate consumption basket is defined by a CES aggregator
as follows:

\[ C_t = \left[ (1-\gamma_1)^{\frac{1}{\eta_1}} C_{H,t}^{\frac{\eta_1-1}{\eta_1}} + \gamma_1^{\frac{1}{\eta_1}} C_{F,t}^{\frac{\eta_1-1}{\eta_1}} \right]^{\frac{\eta_1}{\eta_1-1}} \text{ such that} \]

\[ C_{H,t} = \left[ \int_0^1 C_{H,t}(i)^{1-\frac{1}{\eta_1}} di \right]^{\eta_1}, \quad C_{F,t} = \left[ \int_0^1 C_{F,t}(i)^{1-\frac{1}{\eta_1}} di \right]^{\eta_1} \]

where \( \eta_1 \) is the elasticity of substitution between domestic and foreign goods, \( \gamma_1 \) is the share
of imports in the consumption basket and \( \epsilon \) is the elasticity of substitution across goods within
each category.

\(^3\)Habit formation in preferences enables the estimation of model parameters. Presence of habits generates
impulse responses within the model that correspond to impulse responses to uncertainty shocks in the data.
Also, presence of habits in consumption help in defining steady states that match empirical evidence.
The budget constraint faced by the household is given by:

$$P_t C_t + P_t \Gamma_t + b_t + X_t F^*_t = P_{H,t} W^*_t L_t + \Pi_t + R_{t-1} b_{t-1} + X_t R^*_{t-1} F^*_{t-1}$$  \(1\)

where, the aggregate price index $P_t$ is a CES combination of the price index for domestically produced goods - $P_{H,t}$ and the import price index $P_{F,t}$:

$$P_t = [(1 - \gamma_1)P_{H,t}^{1-\eta_1} + \gamma_1 P_{F,t}^{1-\eta_1}]^{\frac{1}{1-\eta_1}}$$ such that

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right]^{1-\epsilon}, \quad P_{F,t} = \left[ \int_0^1 P_{F,t}(i)^{1-\epsilon} di \right]^{1-\epsilon}$$

$W^*_t$ is the real wage measured in terms of $P_{H,t}$ that households obtain from supplying labor for production of wholesale goods. $R_t$ is the gross nominal rate of interest at home and $R^*_t$ is the gross nominal rate of interest abroad. $X_t$ is the nominal exchange rate\(^4\). Households can invest in domestic bonds: $b_t$ and foreign bonds: $F^*_t$ subject to portfolio holding costs $\Gamma_t$. The costs to holding foreign and domestic assets are modeled following Elekdag, Justiniano and Tchakarov (2006) and given by:

$$\Gamma_t = \frac{\phi_B}{2} \left( \frac{b_t}{P_t} \right)^2 + \frac{\phi^*_F}{2} \left( \frac{X_t F^*_t}{P_t} \right)^2$$

Quadratic costs characterizing portfolio holdings induce stationarity in consumption and stocks of bond holdings. Households choose $\{C_t, b_t, F^*_t, L_t\}$ subject to the budget constraint and the portfolio holding costs. Given, the set-up described above the intra-temporal optimization condition of the households can be described as follows:

$$\frac{L^*_t}{(C_t - hC_{t-1})} = \frac{P_{H,t} W^*_t}{P_t}$$  \(2\)

The Euler equation and the modified uncovered interest parity condition following the optimal choice for asset holdings imply:

$$\left[ 1 + \frac{\phi_B^j b_t}{P_t} \right] = \beta E_t \left[ \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\rho} R_t \right]$$  \(3\)

$$\frac{\phi_B^j b_t}{P_t} - \frac{\phi^*_F^j F^*_t X_t}{P_t} = \beta E_t \left[ \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\rho} \left( R_t / \pi_{t+1} - R^*_t X_{t+1} / \pi_{t+1} \right) \right]$$  \(4\)

\(^4\)Home currency price of one unit of foreign currency
The optimal allocation of expenditure across home and foreign goods imply the following demand functions for goods produced at home and the foreign country respectively:

\[ C_{H,t} = (1 - \gamma_1) \left( \frac{P_t}{P^{*}_{H,t}} \right)^{\eta_1} C_t \]

\[ C_{F,t} = \gamma_1 \left( \frac{P_t}{P^{*}_{F,t}} \right)^{\eta_1} C_t \]

### 3.2 Foreign Sector

Aggregate demand \( (C^*_t) \), aggregate price index \( (P^*_F,t) \) and interest rate \( (R^*_t) \) for the foreign economy (here approximated as rest of the world) are assumed to be constant and treated as parameters in the model. Following Monacelli (2005) and Gertler, Gilchrist and Natalucci (2007), I assume that the Law of One Price holds at the wholesale level for foreign transactions. Price of exports evolves as follows: \( P^*_H,t = \frac{P_{H,t}}{X_t} \). Demand for exports is given as follows:

\[ C^*_{H,t} = \left[ \gamma_2 \left( \frac{P^*_H,t}{P^*_F,t} \right)^{-\eta} C_t \right]^{\rho'} C^*_{H,t}^{1-\rho'} \tag{5} \]

where \( \eta \) is the elasticity of substitution between exports and domestically produced goods in the foreign country and \( \gamma_2 \) is the fraction of home country imports. The parameter \( \rho' \) helps govern the responsiveness of export demand to changes in domestic prices - \( P_{H,t}, X_t \) by scaling the price elasticity of export demand. \( \rho' = 1 \) implies that a one percent change in relative prices leads to a change in export demand by \( \eta \) percent, whereas \( \rho' \in (0,1) \) scales down this effect with the change in demand being given by \( \rho' \eta \) percent.\(^5\) Furthermore, the foreign economy is modeled as a large economy such that imports from the home country constitute a negligible portion of the consumption basket and \( P^*_t \approx P^*_F,t \). That is the CPI in the foreign country is equal to the price of domestically produced goods in the foreign country. I further set \( P^*_F,t = 1 \) while solving the model. This implies that the real exchange rate is defined as follows:

\[ q_t = \frac{X_t P^*_F,t}{P_t} = X_t \frac{P^*_F,t}{P_t} \]

\(^5\)Given that I approximate the foreign sector as rest of the world, \( \rho' \in (0,1) \) enables me to slow down the responsiveness of exports to changes in domestic prices.
3.3 Entrepreneurs

In this paper, and differentiate between advanced and emerging countries in terms of the cost of credit they face in global credit markets. I empirically validate this assumption by examining the country-level credit ratings assigned by Standard and Poor across a sample of 82 countries comprising 32 advanced economies and 50 emerging countries. I use credit ratings as a proxy for the country-specific spread over the risk-free rate ($R^*_t$ in this model). As figure 1 demonstrates emerging countries on average receive a rating between BB+ and BBB, in comparison to advanced countries which receive an average rating between A+ and AA. This observed difference in financing debt can also be attributed in part to country-specific fundamental characteristics such as differences in the degree of financial integration and intermediation across advanced and emerging countries as demonstrated by the financial development index in figure 2. The financial development index is constructed by combining indices measuring financial depth (size and liquidity of markets), access to financial markets (ability of individuals and companies to access financial services), and efficiency of financial markets (ability of institutions to provide financial services at low cost and with sustainable revenues, and the level of activity of capital

Figure 1: Plotting per capita GDP in dollars (x-axis) and country specific credit ratings assigned by Standard and Poor’s for 82 countries - 32 advanced economies and 50 emerging markets (y-axis). Source: International Monetary Fund.
In order to capture this asymmetry, I model borrowing costs faced by entrepreneurs to evolve as a function of a global component and a country specific component. The global component corresponds to the international risk free rate and is constant across countries. The country specific component is defined as an increasing function of leverage. I model the higher borrowing cost faced by emerging countries in international capital markets (as indicated in figure 1) by making borrowing costs more responsive to leverage for emerging countries. In order to capture this asymmetry in the responsiveness of borrowing costs to leverage I use the financial accelerator mechanism outlined in Gertler, Gilchrist and Natalucci (2007) which generalizes the costly state verification approach adopted in Bernanke, Gertler and Gilchrist (1998) to a small open economy DSGE model.

Entrepreneurs in this set up are risk neutral and produce wholesale goods by combining the capital that they own with labor services which they hire from households. Capital required for production is sourced using a combination of net worth \( N_t \) and foreign currency denominated debt \( D^*_t \). Debt contracts are defined for one period. To ensure that entrepreneurs continue to finance capital requirements using a combination of net worth and foreign debt, I assume that entrepreneurs have a finite life with each surviving the next period with probability \( \theta \). Consequently, the expected lifetime of an entrepreneur is given by \( \frac{1}{1-\theta} \). Additionally, the population of entrepreneurs is stationary and exiting entrepreneurs are replaced by new ones.
Each exiting entrepreneur endows the new entrepreneurs with a constant endowment $E$ to ensure that new entrepreneurs have funds to start production. Finally, capital acquired in period $t$ becomes effective for production in period $t + 1$. Entrepreneurs in this framework can thus be interpreted to represent agents conducting non-financial borrowing. A key assumption that will guide the dynamics in this model is the role of foreign currency denominated debt.

In each period $t$, each entrepreneur indexed by net-worth $N^N_t$, chooses capital stock $(K^N_t^{t+1})$ to be used for production in period $t$ and labor $(L^N_t)$ to be combined with capital from previous period $(K^N_t)$ to be used for production of wholesale goods. I start by describing the optimal choice of labor. Each entrepreneur produces wholesale goods using a Cobb-Douglas production function where $\alpha$ denotes the share of capital and $a_t$ is a measure of productivity that is common to all entrepreneurs such that

$$Y^N_{H,t} = a_t(K^N_t)^\alpha(L^N_t)^{1-\alpha}$$ (6)

The optimal choice of labor $(L^N_t)$ given $K^N_t$ and $a_t$ is:

$$\arg \max_{\{L^N_t\}} P^W_t a_t(K^N_t)^\alpha(L^N_t)^{1-\alpha} - P^H_t W_l L^N_t$$

$P^W_t$ denotes the price of wholesale goods. The first order condition with respect to $L^N_t$ implies:

$$a_t \frac{P^W_t}{P^H_t} (1 - \alpha)(\frac{K^N_t}{L^N_t})^\alpha = W^r_t$$

Rewriting in real terms, by using the domestic price index $(P^H_t)$ such that $\varphi_t = \frac{P^W_t}{P^H_t}$:

$$\varphi_t (1 - \alpha) a_t (\frac{K^N_t}{L^N_t})^\alpha = W^r_t$$ (7)

Given constant returns to scale in production of wholesale goods and perfectly competitive labor market, $\frac{K^N_t}{L^N_t} = \frac{C^N_t}{L^N_t}$. The optimal capital labor ratio is therefore independent of entrepreneur specific net-worth.

I next proceed to describe the capital acquisition decision. The demand for entrepreneurial capital depends on the expected return on capital and the expected marginal financing cost. The expected marginal return on capital in period $t$ is the expected gross revenue net of labor costs normalized by the current market value of capital. The expected gross revenue is the sum
of the expected revenue from selling wholesale goods and sale of undepreciated capital. This can be summarized as:

\[
E_t R^K_{t+1} = \frac{\bar{P}_{W,t} a_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t^i + \left(1 - \delta\right)Q_t K_t^N}{Q_{t-1} K_t^N}
\]

\[
E_t R^K_{t+1} = \frac{\alpha \phi t S_{H,t} a_t \left(K_t L_t\right)^{\alpha - 1} + \left(1 - \delta\right)Q_t}{Q_{t-1}}
\]

\[
E_t R^K_{t+1} = \frac{\alpha \phi t S_{H,t} + \left(1 - \delta\right)Q_t}{Q_{t-1}}
\]

I next describe conditions that summarize the marginal financial conditions. I restrict my attention to one period financial contracts that offer lenders a nominal payoff independent of aggregate risk. I consider a form of the contract that is a reduced form representation of the standard debt contract with costly bankruptcy as used in Gertler, Gilchrist and Natalucci (2007). The contract incorporates the possibility of default and subsequently assumes a premium in case of default. The value of the premium will depend on the country specific fundamental characteristics such as quality of financial intermediation, extent of financial integration and access to financial markets as depicted in figure 2. This is analogous to monitoring costs in Bernanke, Gertler and Gilchrist (1999). I assume that this premium (which is a function of country fundamentals) varies inversely with the status of development of a country and captures the asymmetry in borrowing costs demonstrated in figure 1. The debt contract is summarized by the amount foreign currency denominated loans \(D_t\) and interest rate \(R^*_t \Psi(t)\). Here \(R^*_t\) is the international risk free rate and \(\Psi(t)\) is the country specific component. I model

\[
\Psi(t) = k^\nu_t
\]

to be an increasing function of leverage \(k_t = \frac{Q_t K_t}{N_t}\), and \(\nu\) is the elasticity of borrowing costs with respect to leverage. The difference between countries is captured in this model through different values of \(\nu\) such that weaker degree of financial integration (higher monitoring costs) for emerging countries implies \(\nu^{Emerging} > \nu^{Advanced}\). The optimal choice of capital is obtained
by maximizing the ex ante value of entrepreneurial capital $V_{t}^{N,e}$

$$\arg\max_{\{K_{t,1}^N\}} V_{t}^{N,e} = E_{t} \left[ R_{t+1}^{K} Q_{t} K_{t+1}^{N} - R_{t}^{*} k_{t}^{\nu} X_{t+1} D_{t+1}^{N} \right]$$

subject to

$$Q_{t} K_{t+1} = N_{t}^{N} + \frac{X_{t} D_{t}^{N}}{P_{t}}$$

The first-order conditions of this problem, imply the following marginal financing condition:

$$E_{t} R_{t+1}^{K} = R_{t}^{*} k_{t}^{\nu} E_{t} \frac{q_{t+1}}{q_{t}} \text{ where } q_{t} = \frac{X_{t}}{P_{t}} \quad (10)$$

The marginal financing condition captures the external finance premium that arises in equilibrium. This can be related to the financing premium that arises in Bernanke, Gertler and Gilchrist (1999) to cover bankruptcy costs. The equilibrium condition also implies that all entrepreneurs choose the same leverage since equation 10 is independent of entrepreneur specific characteristics. The ex post value of entrepreneurial capital evolves as:

$$V_{t}^{N} = R_{t}^{K} Q_{t} K_{t}^{N} - R_{t}^{k_{t-1}} D_{t-1}^{N}$$

Integrating of over the mass of entrepreneurs, I obtain the aggregate value of entrepreneurial capital:

$$V_{t} = \int_{N} V_{t}^{N} f_{N} dN = \int_{N} \left[ R_{t}^{K} Q_{t} K_{t}^{N} - R_{t}^{k_{t-1}} D_{t-1}^{N} \right] f_{N} dN = \left[ R_{t}^{K} Q_{t} \int_{N} K_{t}^{N} f_{N} dN - R_{t}^{k_{t-1}} \frac{q_{t}}{q_{t-1}} \int_{N} \frac{q_{t}}{q_{t-1}} (Q_{t} K_{t} - N_{t}) \right]$$

$$\int_{N} K_{t}^{N} f_{N} dN - \int_{N} N_{t}^{N} f_{N} dN \right] = \left[ R_{t}^{K} Q_{t} K_{t} - R_{t}^{k_{t-1}} \frac{q_{t}}{q_{t-1}} \int_{N} \frac{q_{t}}{q_{t-1}} (Q_{t} K_{t} - N_{t}) \right]$$

(11)

where aggregate net-worth $N_{t} = \int_{N} N_{t}^{N} f_{N} dN$, and aggregate capital stock $K_{t} = \int_{N} K_{t}^{N} f_{N} dN$. Finally, given that in each period fraction $\theta$ of entrepreneurs survive, aggregate net worth at the end of each period evolves as:

$$N_{t} = \theta V_{t} + (1 - \theta) E \quad (12)$$
where, E is an exogenous constant that ensures that new-born entrepreneurs are endowed with net-worth to start production.\textsuperscript{6} An important consideration that I want to highlight at this point is the balance sheet effect of the real exchange rate. The assumption of foreign currency debt implies that depreciation of the real exchange rate will dampen the value of entrepreneurial capital, decrease the net-worth and subsequently increase leverage both through the marginal financing condition as well as through $V_t$. Thus, depreciation of the exchange rate in period $t$ will imply an increase in the external financing premium in period $t + 1$.

Finally, exiting entrepreneurs consume $C_e^t = (V_t - E)$ after transferring E to the surviving entrepreneurs. Consumption is allocated between home goods and imports such that $C_{eH,t}^t = (1 - \gamma_1)\left( \frac{P_{H,t}}{P_t} \right)^{-\eta_1} C_t^e$ and $C_{eF,t}^t = \gamma_1\left( \frac{P_{F,t}}{P_t} \right)^{-\eta_1} C_t^e$ respectively.

### 3.4 Capital Producers

Capital producers operate in a perfectly competitive environment, purchase undepreciated capital from entrepreneurs and combine them with new investment goods to construct new capital that is available for production in the next period. Capital producers use both domestic and foreign goods for investment such that aggregate investment evolves as follows:

$$I_t = \left[ (1 - \gamma_1)^{\frac{1}{\eta_2}} I_{H,t}^{\frac{\eta_2 - 1}{\eta_2}} + \gamma_1^{\frac{1}{\eta_2}} I_{F,t}^{\frac{\eta_2 - 1}{\eta_2}} \right]^{\frac{\eta_2}{\eta_2 - 1}}$$

with:

$$I_{H,t} = \left[ \int_0^1 I_{H,t}(i) \frac{1}{1 + \epsilon} di \right]^\frac{1}{1 + \epsilon}, I_{F,t} = \left[ \int_0^1 I_{F,t}(i) \frac{1}{1 + \epsilon} di \right]^\frac{1}{1 + \epsilon}$$

where $\eta_2$ is the elasticity of substitution between domestic and foreign goods, $\gamma_1$ is the share of imports in aggregate investment and $\epsilon$ is the elasticity of substitution across goods within each category. The optimal allocation of expenditure across home and foreign goods imply the following demand functions for goods produced at home and the foreign country respectively:

$$I_{H,t} = (1 - \gamma_1)\left( \frac{P_t}{P_{H,t}} \right)^{\eta_2} I_t, I_{F,t} = \gamma_1\left( \frac{P_t}{P_{F,t}} \right)^{\eta_2} I_t$$

\textsuperscript{6}This can be endogenized as managerial wages to entrepreneurs following Bernanke, Gertler and Gilchrist (1999). However for the scope of this analysis this variable does not play any role. Thus to simplify the model, I assume that E is constant. This parameter helps pin down the value of transfers along with the exit rate $\theta$ that is is consistent with a given value of leverage.
The price index for investment is described as a CES combination of the price index for domestically produced goods - $P_{H,t}$ and the import price index $P_{F,t}$:

$$P^I_t = \left[ (1 - \gamma_1)P^I_{H,t} + \gamma_1 P^I_{F,t} \right]^{1/\gamma_1}$$

where,

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(i)^{1-\epsilon} di \right]^{1-\epsilon}, \quad P_{F,t} = \left[ \int_0^1 P_{F,t}(i)^{1-\epsilon} di \right]^{1-\epsilon}$$

Capital production is characterized by adjustment costs following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) such that $S(.) = S(\cdot)' = 0$ in steady state. Producers of capital goods choose investment $I_t$ as follows:

$$\max_{\{I_t\}} E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[ Q_t K_{t+1} - (1 - \delta)Q_t K_t - \frac{P^I_t}{P_t} I_t \right]$$

subject to:

$$K_{t+1} = (1 - \delta)K_t + \left[ 1 - S\left( \frac{I_t}{I_{t-1}} \right) \right] I_t$$

such that $S\left( \frac{I_t}{I_{t-1}} \right) = \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2$  \hspace{1cm} (13)

This leads to the following optimality condition:

$$Q_t \left[ 1 - S\left( \frac{I_t}{I_{t-1}} \right) - S'\left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} \left[ S'\left( \frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}}{I_t} \right]^2 = \frac{P^I_t}{P_t} \hspace{1cm} (14)$$

where $\lambda_t = (C_t - hC_{t-1})^{-\rho}$

3.5 Retailers and the role for nominal rigidities

In the original framework proposed in Bernanke, Gertler and Gilchrist (1999) the role of retailers is primarily to introduce nominal rigidities in the model so as to analyze the scope of policy intervention by the central bank. In the present paper, nominal rigidities play an important role in generating the simultaneous decline in real variables that is characteristic of an uncertainty shock and is well documented in the empirical literature analyzing uncertainty shocks. Furthermore, Basu and Bundick (2017) show that nominal rigidities are essential to guarantee this co-movement in a closed economy model. Additionally, introducing retailers
for imported goods in addition to domestic goods provides flexibility to analyze the responses of macroeconomic variables under different degrees of exchange rate pass through (Monacelli (2005)).

3.5.1 Retailers - Domestic Goods

Following Gertler, Gilchrist, Natalucci (2007) I assume there is a continuum of monopolistically competitive retailers of measure unity. Each of these retailers purchases wholesale goods at price $P_{W,t}$ from the entrepreneurs, differentiates the products slightly and resells the consolidated aggregate as exports to the rest of the world, to households for consumption and to capital producers for production of investment goods. Retailers also incur a fixed cost of production denoted by $K_H$. Fixed costs are chosen such that profits are zero in steady state. Let $Y_{H,t}(j)$ be the output produced by retailer j. Final domestic output is a CES composite of individual retail goods and is given as

$$Y_{H,t} = \left[ \int_0^1 Y_{H,t}(j) \epsilon^{-\frac{1}{\epsilon}} dj \right]^{\frac{1}{1-\epsilon}} - K_H$$

CES preferences in households, capital producers and rest of the world implies that retailer j faces an isoelastic demand given by: $(\frac{P_{H,t}(j)}{P_{H,t}})^{-\epsilon} Y_{H,t}$. Price stickiness is introduced ala Calvo with fraction $(1 - \kappa_H)$ of domestic retailers being able to reset price in each period. The real marginal cost relevant for retailers of goods produced at home is $P_{W,t}$. The optimal rest price $\hat{P}_{H,t}$ is given as follows:

$$\hat{P}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \kappa_H)^s \frac{\lambda_{H,t+s}}{A_t} \Pi_{H,t+s} \frac{P_{W,t+s}}{P_{H,t+s}} Y_{H,t+s}}{E_t \sum_{s=0}^{\infty} (\beta \kappa_H)^s \frac{\lambda_{H,t+s}}{A_t} \Pi_{H,t+s} Y_{H,t+s}}$$

where $\Pi_{H,t+s} = \frac{P_{H,t+s}}{P_{H,t}}$ with the GDP deflator evolving as:

$$P_{H,t}^{1-\epsilon} = \kappa_H P_{H,t-1}^{1-\epsilon} + (1 - \kappa_H) \hat{P}_{H,t}^{1-\epsilon} \tag{15}$$

The other advantage of introducing nominal rigidities via retailers is to eliminate the loss of output due to price dispersion. This simplification helps in reducing the number of state variables in the model and aids the estimation by reducing the computational burden.
3.5.2 Retailers - Imported Goods

For the case of imported goods, I assume incomplete pass through following Monacelli (2005). Retailers of imported goods purchase imports at dock such that PCP (producer currency pricing) holds. However, in setting the domestic price of imports the importers solve a dynamic markup problem characterized by nominal rigidities ala Calvo with fraction \(1 - \kappa_F\) of retailers being able to optimally reset the price in each period. The relevant real marginal cost for retailers of imported goods is therefore \(\frac{X_t P_F^*}{P_{F,t}}\) where \(P_{F,t}\) is the price of imported goods at home and \(P_{F,t}^*\) is the foreign currency price of the wholesale imported goods. Similar to retailers of domestic goods, retailers of imported goods purchase wholesale imported goods, differentiate them slightly and sell the final consumption aggregate of imported goods to households, and capital producers. Retailers of imported goods also incur fixed cost of production denoted by \(K_F\). Fixed costs are chosen such that profits are zero in steady state. Let \(Y_{F,t}(j)\) be the output produced by retailer j. The final imported good is a CES composite of individual retail goods and is given as

\[
Y_{F,t} = \left[ \int_0^1 Y_{F,t}(j)^{\frac{1-\epsilon}{\epsilon}} \, dj \right]^{\frac{\epsilon}{1-\epsilon}} - K_F
\]

CES preferences in households, capital producers and rest of the world implies that retailer j faces an isoelastic demand given by: \(\left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\epsilon} Y_{F,t}\). The optimal rest price \(\hat{P}_{F,t}\) is given as follows:

\[
\hat{P}_{F,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \kappa_H)^s \frac{\Delta_{1-s} \Pi_{F,t+s}}{\Delta_t} \frac{X_t P_{F,t+s}^*}{P_{F,t+s}}}{E_t \sum_{s=0}^{\infty} (\beta \kappa_F)^s \frac{\Delta_{1-s} \Pi_{F,t+s}^*}{\Delta_t} Y_{F,t+s}}
\]

where \(\Pi_{F,t+s} = \frac{P_{F,t+s}}{P_{F,t}}\) with the import price index evolving as:

\[
P_{F,t}^{1-\epsilon} = \kappa_F P_{F,t-1}^{1-\epsilon} + (1 - \kappa_F) \hat{P}_{F,t}^{1-\epsilon}
\]

The parameter \(\kappa_F\) control the degree of exchange rate pass-through in imports in this model - with values of \(\kappa_F\) closer to 0 denoting a scenario that is closer to PCP (producer currency pricing) and values of \(\kappa_F\) closer to 1 denoting a scenario that is closer to LCP (local currency pricing).
3.6 Monetary Policy

In this model, household utility is defined in terms of habit adjusted consumption. The central bank conducts monetary policy taking into account this feature and follows a modified Taylor rule that responds to CPI inflation, output gap as well as output growth. This specification of the Taylor rule is similar to what was adopted in Smets and Wouters (2007).

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{(1-\chi)} \left[ \left( \frac{Y_{H,t}}{Y_H} \right)^\chi \left( \frac{\pi_t}{\pi} \right)^{\chi \pi} \left( \frac{Y_{H,t-1}}{Y_{H,t-1}} \right)^{\chi \Delta \pi} \right]^{\chi}
\]  

(17)

Here \( Y_H \) is the steady state output and \( R_t \) is the gross nominal interest rate and \( \pi_t = \frac{P_t}{P_{t-1}} \).

3.7 Market clearing

Market clearing implies the following resource constraint for the model economy:

\[
Y_{H,t} = \underbrace{\frac{P_t}{P_{H,t}}(C_t + I_t)}_{\text{Domestic Demand}} + \underbrace{\frac{P_{F,t}}{P_{H,t}} Y_{F,t}}_{\text{Net Exports}} + \underbrace{K_H + \frac{P_{F,t}}{P_{H,t}} K_F}_{\text{Fixed Costs}}
\]

(18)

Finally, the model is closed by imposing a market clearing condition for domestic bonds. That is, \( b_t = b \).

3.8 Exogenous Processes

The empirical literature examining the effect of uncertainty shocks on macroeconomic variables typically incorporates a proxy for aggregate uncertainty such that it captures upward surges in uncertainty across different sectors of the economy. The model setup described so far accommodates two sources of exogenous disturbances – shock to household preferences, capturing demand side uncertainty, and shocks to the aggregate productivity process entering through the Cobb-Douglas production, capturing supply side uncertainty. The first moment or the level of aggregate productivity evolves as an AR(1) process with given by:

\[
a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \sigma^a_t \epsilon_t^a
\]

(19)
A shock to $u^a_t$ would correspond a shock to the first moment or a shock to the level of aggregate productivity. Given that uncertainty arises in the model from the time varying volatility of the exogenous disturbances, the key variable of interest is $\sigma^a_t$. $\sigma^a_t$ governs the standard deviation of the aggregate productivity process. Similarly, a shock to $u^z_t$ corresponds a shock to the exogenous processes interacting with household preferences. A shock to preferences can be interpreted to induce demand shocks by making households more impatient.

$$z_t = (1 - \rho_z)\sigma^a_z u_{t-1} + \sigma^a_t u^a_t$$

Analogous to $\sigma^a_t$, $\sigma^z_t$ governs the standard deviation of the shock to preferences. I construct $\sigma^a_t$ and $\sigma^z_t$ to evolve as follows:

$$\sigma^a_t = (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma^a_{t-1} + \eta_{\sigma^a} u^a_t + \eta_{\sigma^C} u^C_t$$

$$\sigma^z_t = (1 - \rho_{\sigma^z})\sigma^z + \rho_{\sigma^z}\sigma^z_{t-1} + \eta_{\sigma^z} u^z_t + \eta_{\sigma^C} u^C_t$$

A shock to $u^a_t$ therefore gives rise to supply side uncertainty only triggering an increase in the volatility of aggregate productivity, whereas a shock to $u^z_t$ gives rise to demand uncertainty only by increasing the volatility of preferences among households. I compare the transmission of shocks to the first and second moment in figure 3. As demonstrated, a shock to the first moment ($u^a_t, u^z_t$) does not change the ergodic distribution of the underlying exogenous process. However, shocks to the second moment ($u^a_t, u^z_t, u^C_t$) alter the distribution of the process under consideration and make extreme events more likely than before.
In order to define the notion of aggregate macro uncertainty within the model such that there is an increase in demand and supply uncertainty simultaneously, I allow $u^{a}_t$ and $u^{z}_t$ to have a common component - $u^{C}_t$. A shock to $u^{C}_t$ is therefore the theoretical counterpart of aggregate macroeconomic uncertainty. Uncertainty shocks in this model can therefore arise from supply side only ($u^{a}_t$), demand side only ($u^{z}_t$) or both ($u^{C}_t$).

$u^{a}_t$, $u^{z}_t$, $u^{C}_t$, $u^{F}_t$ are iid processes distributed normally with mean 0 and standard deviation of 1 respectively. The parameters $\sigma^{a}(\sigma^{z})$, and $\eta^{a}(\eta^{z})$ control the degree of mean volatility and stochastic volatility in aggregate productivity (preferences): with a high $\sigma^{a}(\sigma^{z})$ implying a high mean volatility of aggregate productivity(preferences) and a high $\eta^{a}(\eta^{z})$ implying a high degree of stochastic volatility in aggregate productivity (preferences). Equations 1-22 describe the equilibrium conditions of the model.

3.9 Numerical Solution and Model Calibration

3.9.1 Numerical Solution

The goal of this paper is to explore the interaction of uncertainty shocks and financial frictions in generating business cycle asymmetries across countries. Given that I interpret an uncertainty shocks arising from stochastic volatility of exogenous processes, I need to deviate
from the standard log-linear formulation for the model solution since the first order solution is invariant to the standard deviation of exogenous shocks. A second order solution is not sufficient to generate dynamic effects to an uncertainty shocks since the coefficients on the linear and quadratic terms for the state vector for a second-order expansion of the decision rule are independent of the volatility of the exogenous shocks (Schmidt-Grohe and Uribe 2004). Therefore, I consider a third order approximation to the equilibrium condition to solve the model. The impulse responses are computed using the pruning method suggested in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2016).

3.9.2 Calibration

**Calibrating external finance premium across countries:** In order to emphasize the interaction of borrowing costs and aggregate macroeconomic uncertainty in generating more volatile business cycles in emerging countries vis-a-vis advanced countries, I calibrate the representative models for advanced and emerging countries to differ only on the dimension that governs that spread over the international risk free rate. This is captured by the parameter $\nu$ in the model. I calibrate the parameters such that the leverage is same however the parameter $\nu$ is different. The model is then able to capture the differences in the transmission of an uncertainty shock that is entirely attributed to the cost of credit for calibrations representing advanced and emerging countries respectively. The steady states of the model calibrated for the same level of leverage but different $\nu$ reflects how higher borrowing costs translates into lower values of GDP, consumption and investment. I present a detailed representation of the steady state in section 1.1 in the appendix. Table 1 defines the calibrations for representative advanced and emerging countries. The given values of leverage and $\nu$ imply borrowing costs of 4.76% and 7.18% per period for representative emerging and advanced countries respectively.

**Calibration strategy for exogenous processes:** The volatility of stock market returns

<table>
<thead>
<tr>
<th>Model type</th>
<th>Leverage (k)</th>
<th>Elasticity of borrowing costs wrt leverage ($\nu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representative Advanced Country</td>
<td>2.5</td>
<td>0.04</td>
</tr>
<tr>
<td>Representative Emerging Country</td>
<td>2.5</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Table 1: Calibrating $\nu$
is a commonly used empirical proxy for measuring aggregate macroeconomic uncertainty. I use this to calibrate the mean volatility of productivity and preferences across - $\sigma^x(\sigma^z)$ model calibrations. The average volatility of stock market returns between 1993Q1 – 2014Q4 is 0.014 for Mexico (representative emerging country) and 0.0095 for the United Kingdom (representative advanced country). In order to isolate the role of borrowing costs and demonstrate the effectiveness of the model in capturing the asymmetric effect of uncertainty shocks across representative advanced and emerging countries, I fix the mean volatilities to 0.014 across calibrations (details in table 2) for advanced and emerging countries. However, in section 5 when I estimate the recession specific estimates of borrowing costs I allow this parameter to vary across countries. The parameter $\eta_C, \eta_a$ and $\eta_z$ which capture the extent of stochastic volatility is calibrated such that a one standard deviation shock in the model corresponds to a 1% increase in the standard deviation of productivity (and/or preferences). The AR(1) coefficients are calibrated such that shocks to uncertainty are moderately persistent in the model - this is reflecting the empirical feature that upward surges of uncertainty are relatively short lived.

The remaining behavioral parameters have been calibrated as follows:

**Households:** I fix the discount factor $\beta$ to 0.997, the coefficient of risk aversion $\rho = 2$. Household consumption is characterized by external habits with the parameter $h$ governing the extent of indexation to past consumption. For the first set of results where I compare the strength of the model in generating business cycle asymmetries for calibrations corresponding to representative advanced and emerging countries I set $h = 0.5$. However, in section 4, I estimate the value of this parameter. The implied elasticity of intertemporal substitution is therefore $\frac{1}{\rho^{1-h}} = 0.25$. The Frisch elasticity of substitution $\frac{1}{\psi} = 0.5$. The elasticity of substitution has been calibrated as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Calibrated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^x = \sigma^z$</td>
<td>Mean Volatility</td>
<td>0.014</td>
</tr>
<tr>
<td>$\eta_{\sigma_a} = \eta_{\sigma_z} = \eta_C$</td>
<td>Stochastic Volatility</td>
<td>0.00014</td>
</tr>
<tr>
<td>$\rho_{\sigma^x}$</td>
<td>Persistence: $\sigma^x_t$</td>
<td>0.83</td>
</tr>
<tr>
<td>$\rho_{\sigma^z}$</td>
<td>Persistence: $\sigma^z_t$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence: $a_t$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence: $z_t$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma = \bar{z}$</td>
<td>Mean: Level</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Calibrating uncertainty shocks

*Decreasing the elasticity of labor supply amplifies the impact of uncertainty shocks.
between exports and imports for consumption and investment - $\eta_1$ - is set to 0.89 (following Gertler, Gilchrist and Natalucci(2007))\(^9\). $\eta_2$ - the elasticity of substitution between exports and imports is set to 1 allowing for a greater degree of substitutability for rest of the world relative to the small open economy under consideration. Portfolio holding costs for domestic ($\phi_B$) and foreign assets ($\phi_F$) are set to 0.0009 and 0.009 respectively. The portfolio holding costs in conjunction with the discount factor and steady state level of domestic bond holdings pin down the steady state domestic interest rate. For details refer to table 5 in the appendix.

**Entrepreneurs:** In addition to leverage (k) and elasticity of borrowing costs with respect to leverage ($\nu$), the other parameters that characterize the choices of the entrepreneurs are - $\alpha$ - share of capital in the production function and $\theta$ - the exit rate of entrepreneurs. I fix $\alpha$ to 0.5 (following Gertler, Gilchrist and Natalucci(2007)). I set $\theta$ to 0.915 as estimated by Fernandez and Gulan (2015) for the calibration for a representative emerging country. To preserve symmetry in all dimension excepting $\nu$ I calibrate $\theta$ to 0.915 for the representative advanced country as well.

**Retailers:** In addition to leverage and the elasticity of borrowing costs with respect to leverage , the other parameter that is important in driving the results is the extent of nominal rigidities. I calibrate $\kappa_H = 0.75$ - implying that average duration of $\frac{1}{1-\kappa_H}$ = 4 quarters for domestic firms. The parameter $\kappa_F$ governs the extent of price stickiness for firms selling imported goods. This parameter can also capture the extent of exchange rate pass through given the model specification. Higher values of $\kappa_F$ imply a lower extent of exchange rate pass through. I calibrate $\kappa_F = 0.25$ to demonstrate the initial set of results however, in section 4 I estimate this parameter. The elasticity of substitution across goods within a category (domestically produced and imports) is set to 8 such that in steady state firms experience a mark-up of $\approx 15\%$

**Capital Producers:** The key parameters of interest for capital producers comprise of the elasticity of substitution between domestic goods and imports for investment goods - $\eta_2$, the depreciation rate of capital- $\delta$ and investment adjustment costs $S''(.)$. For simplicity I set $\eta_2 = \eta_1 = 0.89$ - the elasticity of substitution between domestic goods and imports for consumption.\(^{10}\). $\delta$ is calibrated to 0.05. $S''(.)$ is initially calibrated to 6. However in section 5,

\(^9\)Decreasing the elasticity of substitution between exports and imports amplifies the impact of uncertainty shocks.

\(^{10}\)Typically, investment goods exhibit a lower degree of substitution in comparison to consumption goods. Letting the price indices for investment and consumption to display this heterogeneity will amplify the effects of uncertainty shocks.
I estimate this parameter.

### Table 3: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Intertemporal Elasticity of substitution (after adjusting for habits)</td>
<td>8.5</td>
</tr>
<tr>
<td>( h )</td>
<td>Habit</td>
<td>0.65</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Frisch elasticity of labor supply</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_1 )</td>
<td>Elasticity of substitution between home and foreign goods for consumption (Gertler, Gilchrist &amp; Natalucci (2007))</td>
<td>0.89</td>
</tr>
<tr>
<td>( \phi_H, \phi_F^* )</td>
<td>Portfolio Holding Costs</td>
<td>0.00009, 0.00009</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount Factor</td>
<td>0.997</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>Share of home goods in aggregate consumption</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**Foreign Sector**

| \( \eta \) | Elasticity of substitution between home and foreign goods for foreign country (Gertler, Gilchrist & Natalucci (2007)) | 1 |
| \( \gamma_2 \) | Share of goods produced at home - exports for rest of the world | 0.0187 |
| \( C^* \) | Aggregate consumption for rest of the world | 200 |
| \( P^*_F \) | CPI for Rest of the world | 1 |
| \( R^*_G \) | Gross foreign Interest Rate (quarterly) (1.0099 (1.04% Annualized after quarterly compounding)) | 1.0099 |
| \( \rho' \) | Persistence of export demand from rest of the world | 0.25 |

**Entrepreneurs**

| \( \alpha \) | Share of capital in production process | 0.5, Gertler, Gilchrist & Natalucci (2007) |
| \( \theta \) | Exit rate of entrepreneurs | 0.95, Fernandez and Gulan (2015) use 0.9 |

**Capital Producers**

| \( \eta_2 \) | Elasticity of substitution between home and foreign goods for investment | 0.89 |
| \( \delta \) | Depreciation rate | 0.05 |
| \( S'' \) | Elasticity of investment adjustment costs | 6 Smets and Wouters (2007) use 5.74 |

**Retailers**

| \( \epsilon \) | Elasticity of substitution across varieties for domestically produced goods | 8 |
| \( \epsilon_1 \) | Elasticity of substitution across varieties for foreign goods | 8 |
| \( \kappa_H \) | Calvo price stickiness for retailers of domestic goods (Gertler, Gilchrist & Natalucci (2007)) | 0.75 |
| \( \kappa_F \) | Calvo price stickiness for retailers of imported goods | 0.25 |

**Monetary Policy: Taylor Rule Coefficients**

| \( \chi_y \) | Output deviation from steady state | 0.08 - Smets and Wouters (2007) |
| \( \chi_{\Delta y} \) | Output growth | 0.22 Smets and Wouters (2007) |
| \( \chi_{\pi} \) | CPI inflation | 1.5 |

While solving the model I assume that \( C_t^e \) is set equal to zero. This is aligned to the assumption in BGG framework which fixes the share of entrepreneurial consumption to 0.01. This simplification does not alter the transmission of uncertainty shocks in the model.

4 Results

4.1 Transmission Mechanism of an uncertainty shock

Uncertainty shocks in this setup can arise exclusively from demand driven uncertainty (shock to \( u_t^z \)), supply side uncertainty (shock to \( u_t^\alpha \)) and shock to aggregate uncertainty - shock that
simultaneously triggers an upward surge in uncertainty across both sectors of the economy ($u^C_t$). The main difference between a shock to the first moment and an uncertainty shock (shock to the second moment) is that the latter leaves the first moment unchanged and however, changes the shape of the distribution by widening the tails. Therefore, higher uncertainty about future productivity (preferences) induces a precautionary response among the agents in the model by making extreme events more likely than before.

For the scope of demonstrating the transmission mechanism I focus on a one standard deviation shock to the common component between the volatilities of demand and supply as it induces a precautionary savings response among households and precautionary pricing behavior among firms. The model calibration is such that a one standard deviation shock to this common component leads to 1% increase in the volatility of preferences and aggregate productivity respectively.

An uncertainty shock to $u^C_t$ leads to a 1% increase in uncertainty about future productivity as well as future preferences. Given, that a bad shock to productivity is more now likely firms engage in precautionary pricing behavior to hedge against risks of reduced profitability in the future by increasing their mark-up over marginal cost (Born and Pfeifer 2017). This consequently leads to an inward shift of the labor demand curve. The increased mark-up translates to an increase in the price of domestic goods triggering a decrease in consumption and investment demand along with an increase in the marginal utility of wealth. The decrease in consumption demand is amplified as households respond to uncertainty about future preferences by engaging in precautionary savings behavior – reducing consumption demand and increasing labor supply. This leads to an outward shift of the labor supply curve. In equilibrium, wages and hours both decline on impact. The dynamics of labor demand relies crucially on nominal rigidities for retailers of domestic goods and emphasizes the mechanism suggested in Basu and Bundick (2017). Figure 3 illustrates these dynamics.
The reduction in investment demand leads to decline in the price of capital. Given that capital stock remains unchanged, the fall in employment triggers a decline in the marginal productivity of capital and in conjunction with the decline in the price of capital this causes the real rate of return on capital to fall. This decline in the rate of return on capital erodes entrepreneurial net-worth and causes leverage to increase. These dynamics are qualitatively similar across the two calibrations of the model with the calibration corresponding to emerging countries exhibiting an amplified response. (Refer to figures 4 and 5)
The key differentiating feature in responses is brought about by the equilibrium condition that defines the external finance premium. Recall,

\[ E_t^t K_{t+1} = R_t^t \left[ \frac{Q_t K_t}{N_t} \right]^{\nu} \frac{q_{t+1}}{q_t} \]

When the value of \( \nu \) is large enough, the decrease in capital demand triggered by the decrease in investment is not sufficient to counter the increase in leverage. This decline in leverage is brought about by the decrease in the value of entrepreneurial capital. Therefore, to restore equilibrium, the currency depreciates as \( q_t \) increases. The depreciation of domestic currency further erodes the value of entrepreneurial capital and increases leverage. Thus, for \( \nu^{Emerging} > \nu^{Advanced} \), the initial amplification in leverage induced by a higher value of \( \nu \) is further amplified due to the depreciation of the exchange rate. Higher borrowing costs in
addition to foreign currency denominated debt are key channels that generate the amplified responses in leverage, exchange rate and investment for the calibration corresponding to that of a representative emerging country. In addition to reinforcing the financial accelerator, if the depreciation in the real exchange rate offsets the increase in the price of domestic goods relative to the CPI, it triggers an increase in the demand for exports from rest of the world. This is can be seen from the following equation governing export demand:

\[
C_{H,t}^* = [\gamma_2 \left( \frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-\eta} C_{t}^{*\rho} C_{H,t}^{1-\rho*}] - [\eta_2 \left( \frac{P_{H,t}}{X_t P_{F,t}^* / P_{t}} \right)^{-\eta} C_{t}^{*\rho} C_{H,t}^{1-\rho*} - \rho_\star C_{H,t}^{1-\rho*}]
\]

Therefore as long as the increase in \(q_t\) exceeds the decline in \(P_t P_{H,t}\), demand for exports increases in response to an upward surge in aggregate uncertainty. These dynamics are demonstrated in figure 6.

While on one hand a weaker domestic currency propels export demand on the other hand, it amplifies the decline in import demand. Thus, in conjunction, the two can generate an increase in net-exports. For the calibration corresponding to a representative advanced country, this depreciation of the real exchange rate is absent. Consequently, the calibration does not generate this countercyclical response in trade balances. The model calibrations differing only with respect to this one parameter \(\nu\) is not only able to generate the asymmetric response in real variables to uncertainty shocks, with larger values of \(\nu\) leading to amplified decline. It is also able to generate the strong countercyclicality in trade balances that is the key distinguishing feature between business cycles in advanced and emerging countries.
Finally, given the decline in consumption and investment demand exceed the increase in net-exports and overall GDP declines. The model specification successfully generates the simultaneous decline in consumption investment and GDP along with countercyclical trade balances for model calibration corresponding to an emerging country. Furthermore, the model can produce the asymmetry in the responses of real variables to an uncertainty shock across model calibrations for advanced emerging countries. (Refer to figures 7 and 8 respectively)
5 Estimating the role of financial frictions across countries during recessions in generating business cycle asymmetries

The results so far underscore the interaction of uncertainty shocks and financial frictions in generating business cycle asymmetries across advanced and emerging countries. Empirical evidence\textsuperscript{11} on the impact of uncertainty shocks on real variables suggest that the effects of macroeconomic uncertainty are largely countercyclical. That is, upward surges in macroeconomic uncertainty matter more during downturns in business cycles.

One of the ways to test this interaction between financial frictions and uncertainty shocks could be to estimate the nonlinear model solved using third order approximation across different regimes - recessions and expansions. However, this approach is computationally burdensome. Instead, of estimating regime specific responses across countries, I use a modified version of the VAR-based impulse response function matching estimator. I estimate the role of financial frictions and uncertainty shocks in recessions by matching recession specific responses of macroeconomic variables to uncertainty shocks generated from a Smooth Transition Vector Auto Regression Model (STVAR) to the impulse responses of relevant variables to uncertainty shocks generated from the theoretical model described in the earlier section.

5.1 Recession specific impulse responses from the STVAR model

The empirical evidence examining the effect of uncertainty shocks across advanced and emerging countries (Swallow and Cespedes (2013)) suggests that emerging countries are more responsive to uncertainty shocks with consumption and investment recording sharper declines and weaker recoveries. Using the STVAR model, and the U.K and Mexico as representatives of open and advanced, and open and emerging countries, I demonstrate that the recession specific response of real variables to uncertainty shock is larger and more persistent for emerging countries in contrast to advanced countries.\footnote{In an earlier paper, I estimate the STVAR model for the U.S., the U.K and Mexico.}

The STVAR model distinguishes between a recessionary regime and a ‘catch all’ non-recessionary regime. The model also, incorporates the ability to allow for country specific differences in guiding the smoothness of transition across regimes. The detailed model specification is given below:

\[
Y_t = F(z_{t-1})B_R(L)Y_t + (1 - F(z_{t-1}))B_{NR}(L)Y_t + \epsilon_t
\]  

(23)

\[
\epsilon_t \sim N(0, \Omega_t)
\]  

(24)

\[
\Omega_t = F(z_{t-1})\Omega_R + (1 - F(z_{t-1}))\Omega_{NR}
\]  

(25)

\[
F(z_t) = \frac{exp(-\gamma z_t)}{1 + exp(-\gamma z_t)} \text{ and } \gamma > 0
\]  

(26)

\[
E(z_t) = 0 \text{ and } Var(z_t) = 1
\]  

(27)

\[Y_t = [U_t, I_t, C_t, TB_t, \Pi_t, r_t]'\] is the baseline specification of endogenous variables where \(U\) is the country specific proxy for ‘aggregate macroeconomic uncertainty’, \(I\) is the growth rate of investment, \(C\) is the growth rate of consumption, \(TB\) is the first difference of net exports expressed as a percentage of GDP, \(\Pi\) is the inflation and \(r\) is the policy rate. I estimate the baseline specification for each country. I quantify uncertainty by using the volatility of stock
market returns. I have constructed the quarterly measure of country specific uncertainty by averaging the monthly standard deviation of stock market returns calculated using daily data. Volatility of stock market returns is standard measure of macro-financial uncertainty, however, Bloom (2014) demonstrates that measures such as the VIX, standard deviation of stock market returns are correlated with other measures of macroeconomic uncertainty and can be used to represent aggregate macroeconomic uncertainty as well.

As described in the model specification, the STVAR framework allows for a two-way propagation mechanism for shocks to uncertainty. The regime specific VAR coefficients defined by \( \{B_R, B_{NR}\} \) allow for dynamic propagation of shocks and the regime specific variance covariance matrices \( \{\Omega_R, \Omega_{NR}\} \) allow for contemporaneous propagation of uncertainty shocks. \( \{B_R, \Omega_R\} \), therefore, describes the behavior of the economy deep in recessions and likewise, \( \{B_{NR}, \Omega_{NR}\} \) describes the behavior of the economy during ‘catch all’ non-recessionary phases.

The parameter \( \gamma > 0 \) governs the smoothness of transition from recessionary to the non-recessionary regime. As \( \gamma \rightarrow \infty \) the transition becomes very abrupt between the regimes, whereas setting \( \gamma = 0 \) reverts the system to the linear VAR specification. I set \( \gamma = 1.75 \) for the U.K and \( \gamma = 2.5 \) for Mexico to capture the differences in volatilities exhibited by key macro variables across the two countries. The variable \( z_t \) governs the transition from one regime to the other. The goal is to capture the differences in business cycles across countries by appropriately calibrating \( \gamma \) and choosing the state transition variable such that the system spends sufficient time in recessions. In the current set up \( F(z) \) is given by the logistic function. It defines the likelihood of being in any particular state, with \( F(z) \approx 1 \) implying the recessionary regime and \( F(z) \approx 0 \) implying the expansionary regime. The logistic function is used for assigning regime specific probabilities by using the smoothness parameter \( (\gamma) \) and the state transition variable \( (z_t) \) as inputs.

Following Auerbach and Gorodnichenko (2012) the transition function enters the VAR specification (equation 1) with a lag of one period to avoid contemporaneous effects of policy variables in defining the state of the economy. The state transition variable is not included in the system of endogenous variables, thus, eliminating interaction and feedback effects between the state transition variable and the dynamics of the macroeconomic variables included in the system. The choice of the transition function is very important as this is the driving force that induces
non-linearities in endogenous variables at turning points in the business cycle. While there are multiple ways to capture regime switches in the business cycle, following Auerbach and Gorodnichenko (2012) (and what was adopted in Caggiano, Castelnuovo and Groshenny 2014), I have defined $z_t$ to be the standardized 7 quarter moving average of the growth rate of real GDP. Therefore, $z_t > 0$ implies that the growth trajectory of real GDP is above average and vice versa.\footnote{Using a standardized estimate of $z_t$ helps in eliminating scale dependence of $z_t$.}

Given the above model specification, I compute impulse responses of consumption and investment to a 1% shock to uncertainty for the recessionary regime of the STVAR model. This comparison across countries is displayed in figure 3. These have been computed by assuming that there is no feedback from changes in the state transition variable into the dynamics of the macroeconomic variables. This implies that the economy could spend a very long time in a recessionary regime. However, the estimated parameters for the recessionary regime of the STVAR model implies that growth rate of real variables return to zero within 10 quarters of the across countries. When calculated using levels, this implies that the level remains constant after 10 periods of the shock. In essence, what the impulse response records is the transition of the economy from a peak in a business cycle to the trough.

5.2 Impulse Response Function Matching Estimator (IRFME)

The impact of an uncertainty shock on macroeconomic variables is typically characterized by the simultaneous decline in consumption, investment and GDP. Therefore, while estimating the role of financial frictions in generating business cycle asymmetries across countries, I attempt to match the responses of consumption and investment (figure 3). I exclude GDP from the STVAR since, the seven quarter moving average of real GDP growth rate is used as an input in defining the regime specific probabilities. Including, real GDP as a variable in the STVAR specification would imply that the regime changes maybe induced by changes in uncertainty. While this is an interesting question in itself, the main point of focus in this section is to isolate the impact of upward surges in uncertainty during recessionary episodes and quantify the strength of the financial frictions channel in generating the heterogeneous response to uncertainty shocks across countries.

Finally, a comment on the ordering of variables - the impulse responses to a 1% shock to
uncertainty have been constructed with uncertainty ordered as the first variable in the STVAR. This means that the one step ahead forecast error in ‘country specific uncertainty’ is attributed in entirety to uncertainty shocks. Within the STVAR specification the impulse responses of consumption and investment should be interpreted as the maximum response for uncertainty shocks to real variables. This ordering however matches the formulation in the theoretical model described in section 3, where uncertainty is interpreted as the time varying volatility of the process governing the evolution of aggregate productivity and preferences. The approach is similar to what has been adopted in Basu and Bundick (2017) where an upward surge in uncertainty is causally prior to the responses of macroeconomic variables. I proceed to defining the Impulse Response Function Matching Estimator (IRFME) following Hall, Inoue, Nason and Rossi (2012) that helps isolate the role of financial frictions and non-financial leverage during recessions. Let, \( \gamma \) denote impulse responses generated from the DSGE model such that,

\[
\gamma = g(\hat{\phi}, \overline{\phi}, h)
\]

Let \( n \) denote the total number of parameters in the model and \( \hat{\phi} = [\hat{\phi}_1, \ldots, \hat{\phi}_{n_1}] \) denote the subset \( n_1 < n \) parameters that I estimate using the IRFME procedure. \( \overline{\phi} = [\overline{\phi}_{n_1+1}, \ldots, \overline{\phi}_n] \) denotes the set of calibrated parameters in the model. Let \( \hat{\gamma} \) denote the impulse responses to a 1% uncertainty shock constructed using the parameters characterizing the recessionary regime of the STVAR model. \( \hat{\gamma} \) therefore corresponds to the estimate of \( \gamma \). The IRFME of \( \hat{\phi}_i = \hat{\phi}_i(\overline{\phi}, h) \) \( \forall i \in 1, \ldots, n_1 \) such that:

\[
\begin{pmatrix}
\hat{\phi}_1(\overline{\phi}, h) \\
\hat{\phi}_2(\overline{\phi}, h) \\
\vdots \\
\hat{\phi}_{n_1}(\overline{\phi}, h)
\end{pmatrix} = \underset{\hat{\phi}_1(\overline{\phi}, h), \ldots, \hat{\phi}_{n_1}(\overline{\phi}, h)}{\arg \min} \left[ \hat{\gamma} - g(\hat{\phi}, \overline{\phi}, h) \right] \Omega_T(h) \left[ \hat{\gamma} - g(\hat{\phi}, \overline{\phi}, h) \right]
\]

### 5.3 Results of the IRFME procedure

The results suggest that recessions are characterized by higher financial frictions as well as elevated levels of uncertainty. The estimated values of \( \nu \) imply a differential of 4.33% for a given level of leverage (2.5) between the United Kingdom and Mexico during recessions. Moreover,
<table>
<thead>
<tr>
<th>Parameter $\phi$</th>
<th>Definition</th>
<th>Mexico</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>Elasticity of borrowing costs wrt leverage</td>
<td>0.1275</td>
<td>0.085</td>
</tr>
<tr>
<td>$\sigma_{\pi} = \sigma_z$</td>
<td>Average uncertainty</td>
<td>0.044</td>
<td>0.04</td>
</tr>
<tr>
<td>$h$</td>
<td>Persistence of external habits</td>
<td>0.574</td>
<td>0.55</td>
</tr>
<tr>
<td>$S''$</td>
<td>Investment adjustment costs</td>
<td>6.051</td>
<td>6</td>
</tr>
<tr>
<td>$\kappa_F$</td>
<td>Degree of exchange rate pass through</td>
<td>0.544</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>CRRA</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Frisch elasticity of labor supply</td>
<td>9.090</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4: Recession Specific estimates of the key behavioral parameter across countries.

while the level of volatility is higher than average for both countries, the average volatility is 10% higher for Mexico whereas borrowing costs are about 50% higher vis-a-vis the United Kingdom. These results emphasize the role of country fundamentals vis-a-vis exogenously different shocks towards characterizing the excess volatility of real variables in emerging countries.

In addition to elevated uncertainty and amplified financial frictions, estimates of $\kappa_F$ for both countries suggest that nominal rigidities are important for imports as well. Given that $\kappa_F$ captures the degree of exchange rate pass through, these estimates suggest evidence against complete pass through of nominal exchange rate for both countries. Finally, estimates of the Frisch elasticity suggests weaker responsiveness of labor supply in emerging countries. This feature is consistent with the business cycle properties of emerging countries. I compare the impulse responses generated using the estimated parameters summarized in table 4 vis-a-vis the impulse responses from data generated using the STVAR model in figures 9 and 10 respectively.
6 Conclusion

In this paper, I use a benchmark small open economy DSGE model augmented with financial frictions to demonstrate that upward surges in aggregate macroeconomic uncertainty leads to a simultaneous decline in consumption, investment and GDP across model specifications for advanced and emerging countries alike. I explain the excess volatility of real variables in emerging markets vis-a-vis the financial accelerator mechanism that amplifies the increase in borrowing costs and the depreciation of the real exchange rate. This amplification leads to a larger decline
in macroeconomic variables and provides a possible explanation towards the asymmetry in the response of real variables to uncertainty shocks across advanced economies and emerging countries. I assess the role of the financial frictions channel along with key behavioral parameters in amplifying the response of real variables in response to an uncertainty shock during recessions. I find that the cost of credit differs significantly between advanced and emerging countries during downturns in the business cycle. The results from estimation emphasize the role of country fundamentals vis-à-vis exogenously different shocks towards characterizing the excess volatility of real variables in emerging countries.
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Appendix

1 Steady State Properties

1.1 Characterizing the non-stochastic steady state in the model

I derive the non-stochastic steady state of the model such that CPI inflation \( \pi = \frac{P_t}{P_{t-1}} = 1 \), domestic inflation \( \pi_H = \frac{P_{H,t}}{P_{H,t-1}} = 1 \), import price inflation \( \pi_F = \frac{P_{F,t}}{P_{F,t-1}} = 1 \). This will imply that in steady state \( \pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = 1 \) and \( \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} = 1 \) since

\[
\pi_{H,t} = \left[ \kappa_H + (1 - \kappa_H)\pi_{H,t}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \\
\pi_{F,t} = \left[ \kappa_F + (1 - \kappa_F)\pi_{F,t}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}
\]

Substituting these values, I solve for the steady state values of the real marginal cost for production of domestic goods \( \varphi \) and the real marginal cost faced by the retailers of imported goods \( \Psi_F \).

\[
\varphi = \frac{\epsilon - 1}{\epsilon} \\
q = \frac{\epsilon - 1}{\epsilon} S
\]

where \( S \) is the steady state terms of trade and \( S_H \) is ratio of the CPI to the GDP Deflator

For a given value of leverage \( \frac{K}{N} \), and assuming in steady state \( L = 1 \), I solve the following system of nonlinear equations to solve for the capital \( (K) \), net-worth \( (N) \), consumption \( (C) \), wages \( (W^r) \) and the terms of trade \( (S) \).

\[
S_H = [\kappa_H + (1 - \kappa_H)S^{1-\epsilon}]^{\frac{1}{\epsilon-1}} \\
K \frac{L}{C} = \left[ S_H(k^\epsilon R^* - (1-\delta)) \right]^{\frac{1}{\alpha-1}} \\
W \frac{P_H}{F} = W^r = (1 - \alpha)\epsilon - 1 \epsilon \left( \frac{K}{L} \right)^\alpha \\
C \frac{L}{C} = \left[ \frac{W^r S_H}{P_H} \right]^{\frac{1}{\kappa}} \\
\left( \frac{K}{L} \right)^\alpha (1 - \frac{1}{\epsilon}) = \left[ (1 - \gamma)((S_H)^\eta (C + \delta K)) \right] + \gamma \left( \frac{\epsilon - 1}{\epsilon} S \right) \eta C^* 
\]
<table>
<thead>
<tr>
<th>Variable</th>
<th>k=2.5, ν=0.035</th>
<th>k=2.5, ν=0.05</th>
<th>k=2.5, ν=0.06</th>
<th>k=2.5, ν=0.075</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>5.4048</td>
<td>4.8684</td>
<td>4.5753</td>
<td>4.2055</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.2931</td>
<td>3.1887</td>
<td>3.1296</td>
<td>3.0529</td>
</tr>
<tr>
<td>Investment</td>
<td>1.4606</td>
<td>1.1851</td>
<td>1.0466</td>
<td>0.8843</td>
</tr>
<tr>
<td>Net-Worth</td>
<td>11.6846</td>
<td>9.4807</td>
<td>8.3732</td>
<td>7.0744</td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>0.73579</td>
<td>0.67187</td>
<td>0.635</td>
<td>0.58662</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.73819</td>
<td>0.70159</td>
<td>0.6797</td>
<td>0.65002</td>
</tr>
<tr>
<td>Real Wage</td>
<td>2.3646</td>
<td>2.1299</td>
<td>2.0017</td>
<td>1.8399</td>
</tr>
<tr>
<td>Rate of Return: Capital</td>
<td>1.1826</td>
<td>1.2494</td>
<td>1.296</td>
<td>1.3693</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>1.0416</td>
<td>1.0439</td>
<td>1.0451</td>
<td>1.0465</td>
</tr>
</tbody>
</table>

Table 5: Comparing the steady state values for different values of the elasticity of borrowing costs with respect to leverage ($\nu$) and leverage $k$.

1.2 Pinning down $E$ as a function of model parameters and the steady state leverage

Given, leverage $k$, and capital $K$ (from previous section) I can pin down the steady state level of net-worth $N$. The steady state value of entrepreneurial capital $V_t$ is given by:

$$
V = [r^K - \Psi(k) R^*]K + \Psi(k) R^* N 
$$

$$
V = [r^K - k^\nu R^*]K + k^\nu R^* N 
$$

$$
V = [k^\nu R^* - k^\nu R^*]K + k^\nu R^* N 
$$

$$
V = k^\nu R^* N = r^K N 
$$

Using the equation that characterizes the evolution of net worth after accounting for fraction $(1 - \theta)$ of exiting entrepreneurs:

$$
N = \theta V + (1 - \theta)E 
$$

$$
N = \theta r^K N + (1 - \theta)E 
$$

$$
(1 - \theta r^K)N = (1 - \theta)E 
$$

$$
D = \frac{(1 - \theta r^K)N}{(1 - \theta)} 
$$
Thus, given \( N, \theta, r^K \) I can pin down the value of \( E \) that is consistent with a steady state leverage of \( k \). I want to point out that:

\[
\frac{\partial D}{\partial \theta} = \frac{-N(1 + r^K)}{(1 - \theta)^2} \Rightarrow \frac{\partial D}{\partial \theta} < 0
\]

\[
\frac{\partial D}{\partial \nu} = -\frac{\theta N}{(1 - \theta)} + \frac{1 - \theta r^K}{1 - \theta} \frac{\partial K}{\partial \nu}, \frac{\partial K}{\partial \nu} < 0 \Rightarrow \frac{\partial D}{\partial \nu} < 0
\]

Thus for larger values of \( \nu \) as \( r^k \) increases, \( D \) decreases and may become negative. Therefore, the calibration of \( \theta \) takes into account this dynamic and ensures that \( D > 0 \).