Fiscal Dominance and the Optimal Maturity Structure of Sovereign Debt *

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Abstract

This paper investigate how fiscal dominance shapes the maturity structure of debt. We develop a simple theoretical model and introduce a novel channel of fiscal dominance, illustrating its impact on the maturity structure of debt. We demonstrate how the maturity structure can serve as a conduit for translating an immediate increase in prices into a gradual buildup of future price levels, contingent on specific parametric conditions involving term premium and rollover risks. To empirically validate the results, we compile a comprehensive granular-level dataset on Indian public debt, comprising central government security-level data from 1999 to 2022. We employ the decomposition methodology proposed by Hall and Sargent (2011) to explore how fiscal dominance influences the maturity structure of debt. Our findings reveal an inverse correlation between short-term interest rates and average debt maturity, accompanied by a positive association between average debt maturity and the debt-GDP ratio. We underscore the significance of interest rates and inflation in the management of debt dynamics. Our paper contributes to understanding the role of debt maturity in monetary-fiscal policy coordination, especially in emerging markets , and introduces the 'maturity structure channel of fiscal dominance'.

Key Words: Sovereign Debt, Debt Decomposition, Fiscal Dominance, Term Premium, Debt Management, Fiscal-Monetary Coordination

JEL Codes: : E43, E62, E65, E52, E4, G12, G28, H63

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1 Introduction

In recent years, the global economy has witnessed a substantial increase in sovereign debt-GDP ratios, a trend exacerbated by events such as the Great Financial Crisis and the COVID-19 pandemic. In advanced economies, debt levels surged due to pandemic-related stimulus measures, with the debt-GDP ratio reaching around 125% in 2021, a significant increase compared to pre-pandemic levels (International Monetary Fund (2021)). Similarly, other emerging market economies, including India, also experienced a surge in debt, primarily driven by increased borrowing to address the economic challenges posed by the pandemic. As per IMF estimates, India's debt-GDP ratio has been on the rise, reaching approximately 90% in 2021 (See Figure 1), reflecting the impact of increased borrowing amid the COVID-19 pandemic[1]. This surge in debt is attributed to fiscal measures and stimulus packages implemented to mitigate the pandemic's economic fallout, resulting in a fiscal deficit of around 9.3% of GDP in 2021-22 (World Bank (2021)). Addressing this rise in indebtedness poses a critical challenge for sustaining economic stability and growth, emphasizing the need for prudent fiscal *and* monetary policy intertwining in the years ahead.

In this paper, we analyze the impact of monetary-fiscal policy coordination, or the lack of it, on the maturity structure of debt iin India, a large emerging market economy. There are several reasons why the maturity structure of debt is important including indicating fiscal stress of the government. Notably, during crises, sovereigns often increase the issuance of short-term debt, as observed in previous studies (Ali Abbas et al. (2011); Chen et al. (2019)). This characteristic of debt has implications for the interest burden faced by the government (Greenwood and Vayanos (2014)). We show that the maturity structure of debt becomes important for the central bank, which also serves as the public debt manager of the government[2].This significance becomes particularly pertinent when the central bank's inflation targeting objective conflicts with that of an inflexible or "dominant" fiscal authority concerning the maintenance of the real value of debt at a sustainable level. Through a well-devised debt management policy, the central bank can strategically shift the burden of debt repayment to the future while simultaneously raising interest rates to curb inflation rising out of increased fiscal expenditure. When the debt comes up for repayment in the future, then use *future inflation* as a tool to erode the real market value of debt[3].

[1]General government(Centre + State) debt-GDP ratio.

[2]This phenomena is quite common in many emerging market economies including India. See Stella and Lonnberg (2008), and Demirgüç-Kunt et al. (2013)

[3]This phenomena is also explained in Cochrane (2022a) which shows that how the maturity structure of debt can lead to a gradual rise in prices rather than a sudden one time price level jump. Our analysis draws on this argument but unlike him, we use the market value of debt in analyzing the role of maturity structure. Additionally, through a theoretical model, we explicitly illustrate the intricate interplay between the maturity structure of debt and fiscal dominance in explaining this "maturity structure" channel.

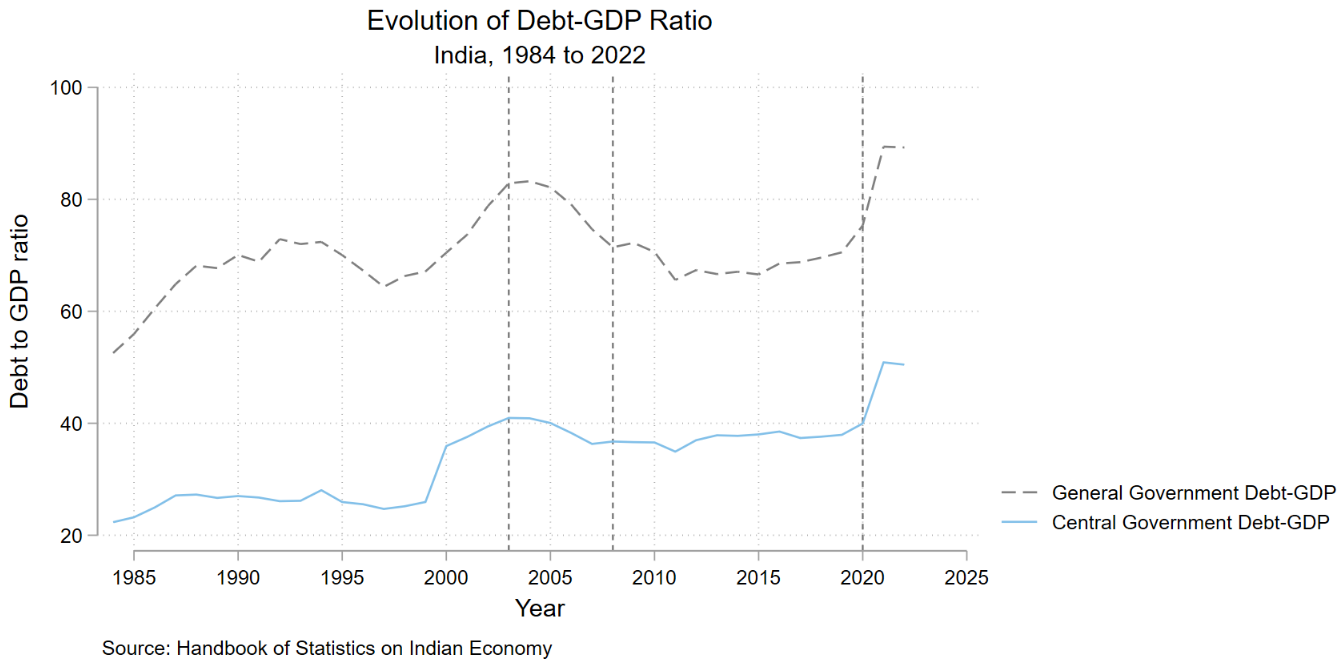

Figure 1: Evolution of Debt-GDP ratio, India

We revisit the subject of fiscal/monetary interdependence in the context of surging debt-GDP ratio post the Global Financial Crisis in 2007 which was exacerbated by the COVID-19 pandemic. To maintain debt at a sustainable level, coordinated efforts between the fiscal and monetary authorities are essential (Kumhof et al. (2010)).

Specifically, we explore whether a monetary authority's commitment to aggressively combat inflation is sufficient to ensure debt stability. The answer to this question depends on the degree of coordination between the monetary and fiscal authorities. If the monetary authority responds to high inflation by increasing interest rates, and the fiscal authority reduces its spending or takes measures to boost budgetary surpluses, both price and debt stability can be maintained.

However, if the central bank's inflation objective collides with an inflexible or dominant fiscal policy stance that is unable or unwilling to adjust primary surpluses to stabilize government debt, then the answer to our question is negative. This latter phenomena, where the fiscal authority is dominant, is termed as *fiscal dominance* (Leeper (1991), Cochrane (2001), Kumhof et al. (2010), Bianchi et al. (2020)).Under a fiscally dominant regime, debt management becomes important due to the opposing stances taken by the authorities and emerges as a central concern for the sovereign authority in balancing fiscal sustainability with economic stability.

When the central bank is mandated to maintain price stability, then under a fiscally dominant regime, it cannot use the current price level as a channel to keep debt at a sustainable level , rather we show that the maturity structure of debt becomes a prime channel for debt sustainability[4]. This channel, using the maturity structure of debt, does not substitute for the inflation channel or the financial repression of fiscal dominance (Sargent et al. (1981), Leeper (1991), and Woodford (2001)); rather, it complements that channel by making future inflation an important factor for debt sustainability.

Why is the maturity structure of debt important for monetary/fiscal coordination in the Indian context? Firstly, India has implemented economic reforms that impose limits on inflation and the deficit, thanks to Flexible Inflation Targeting (FIT) and the Fiscal Responsibility and Budget Management Act (FRBM)[5]. Secondly, an aspect that is unique to India, and for some other EME central banks, is that the central bank is the debt manager of the government. Consequently, the maturity structure of debt carries significant implications for monetary policy (Pandey and Patnaik (2017)). Thirdly, as we show in our stylized facts,the maturity structure of debt in India has been on the rise over the past decade, indicating the government's stance towards preferring for longer maturities. These factors underscore the importance of studying the relationship between the maturity structure of debt and monetary-fiscal coordination in India, especially as its debt-GDP ratio has been on the rise (see Figure 1).

We build a theoretical model of government debt maturity based on He and Xiong (2012) and Greenwood et al. (2015) where the debt management office (DMO henceforth) chooses the optimal maturity of debt to be issued on behalf of the government taking into consideration the trade-off to issue securities at the lowest cost and to manage the "fiscal risks" arising from issuing more debt. We set our model in partial equilibrium in order to focus on the debt management problem of the DMO and introduces a parameter that quantifies the degree of impact from monetary-fiscal coordination, or the lack thereof, on the DMO's choices. We show that the optimal maturity choice depends on four factors: term premium, rollover risks, extent of monetary-fiscal coordination, and the level of debt-GDP.

In order to focus on how fiscal dominance influences the real market value of debt via

[4] If the central bank is mandated to keep inflation under check, then in a fiscally dominant regime it might not be able to use inflation to keep the real value of debt at a lower level. If the central bank issues nominal debt, then the only way to have nonexplosive debt in the face of dominant fiscal policy is to have falling real interest rates when inflation rises(Cochrane (2001), Woodford (2001)), but this goes against the mandate of keeping inflation at check. For more details, see Leeper and Walker (2012), Bianchi (2020), and Cochrane (2022b).

[5] The FIT was adopted in 2014 by the Central Bank of India (Reserve Bank of India) that commits to keep inflation at $4\% \pm 2\%$. FRBM Act was adopted in 2003 to limit the fiscal deficit as a share of GDP at 3% of GDP.

the debt maturity structure, we conduct comparative statics using our model. We demonstrate how changes in the debt level and the short-term interest rate, determined by the central bank, affect the DMO's choice of optimal debt maturity structure. This choice varies depending on whether the economy is operating under a fiscally dominant or a monetary dominant regime.

Our main result is that, under a fiscally dominant regime, there exists a positive correlation between the average maturity of debt and the debt-GDP ratio. Conversely, we observe a negative correlation between the short-term interest rate and the average maturity of debt. We attribute the former relationship to the strategy of postponing repayments into the future while accumulating higher deficits as debt. The latter relationship is explained by the DMO's action of "locking in" the current lower long-term rate when anticipating higher future short-term rates, thereby reducing government interest costs while postponing repayments. Ultimately, as the debt matures, future inflation can be utilized to restore the real value of the debt.

To empirically validate our hypothesis regarding the maturity structure channel of fiscal dominance and the results from our model, we assemble a novel granular level dataset on Indian public debt. Building upon Das and Ghate (2022), our dataset encompasses central government securities at the security level, spanning from 1999 to 2022. Specifically, we focus on marketable dated central government securities, tracking each security's trajectory throughout its lifespan[6] . To delve comprehensively into India's debt dynamics, we meticulously track the trajectory of each security throughout its lifespan. Our analysis centers on the market value of outstanding debt, calculated using the yield to maturity (YTM) methodology based on Hall and Sargent (2011), abbreviated as HS. To analyze the role played by the maturity structure of debt and other important factors affecting debt dynamics—namely, inflation, nominal interest rates, and the economy's growth rate—we conduct a decomposition exercise based on the HS methodology which enables us to delineate the individual contributions of these key variables to India's debt dynamics.

Our analysis reveals a consistent trend of increasing average debt maturity throughout the sampled period (Figure 2). This observation aligns with the government's reports on average maturity derived from aggregate debt figures in their budgetary documents, thus corroborating the initial aspect of our proposed channel, which emphasizes the expansion of the debt maturity structure. The results from our decomposition exercise underscore

[6] Using marketable debt is important for our purpose since our channel of fiscal dominance works through changing the market value of debt by changing the market price. This approach not only considers the market price but also explicitly factors in capital gains and losses, influenced by a range of variables including investor sentiment, security risk profiles, and liquidity dynamics (Hall and Sargent (2011)). For more details see Campbell et al. (2016), and Gagnon et al. (2018).

the significant influence of nominal interest rates, especially the short-term interest rate, in shaping the dynamics of debt in the Indian context.

Furthermore, upon analyzing the outcomes according to distinct maturity segments, we observed that holders of long-term bonds encounter adverse real returns. Notably, the inflation factor emerges as pivotal in reducing the debt-GDP ratio, particularly when considering longer-maturity debt These findings underscore the intricate interplay of factors in India's debt dynamics, highlighting the importance of interest rates and inflation across various debt maturity segments. This emphasizes the significance of the maturity structure in transferring inflation to long-term bondholders and mitigating the current increase in prices required to maintain the real value of debt.

Our analysis establishes a positive correlation between the debt-GDP ratio and the average maturity of debt and a negative correlation between the short-term rate and average maturity. These two findings, in conjunction with the role of inflation and short-term interest rates revealed through the decomposition, validate the results of our theoretical model. They demonstrate that fiscal dominance impacts the maturity structure of debt by increasing it and using future inflation as a tool to maintain the real value of debt at a sustainable level.

Furthermore, while this study primarily focuses on India, its findings hold relevance for a broader spectrum of emerging market economies (EMEs). Many EMEs share similar characteristics in terms of fiscal challenges, monetary policy dilemmas, and the management of sovereign debt. The issues of fiscal dominance, debt sustainability, and the interplay between fiscal and monetary authorities are not unique to India but are prevalent concerns in other EMEs as well. The insights from our paper can help in the understanding of how debt maturity impacts fiscal and monetary policy coordination becomes increasingly important for the stability and growth of EMEs.

The remainder of this paper proceeds as follows. Section 2 provides a review of the literature. Section 3 presents some stylized facts about the debt dynamics in India. Section 4 presents the static model. Section 5 and 6 present the data and accounting decomposition steps along with the decomposition results and section 7 concludes.

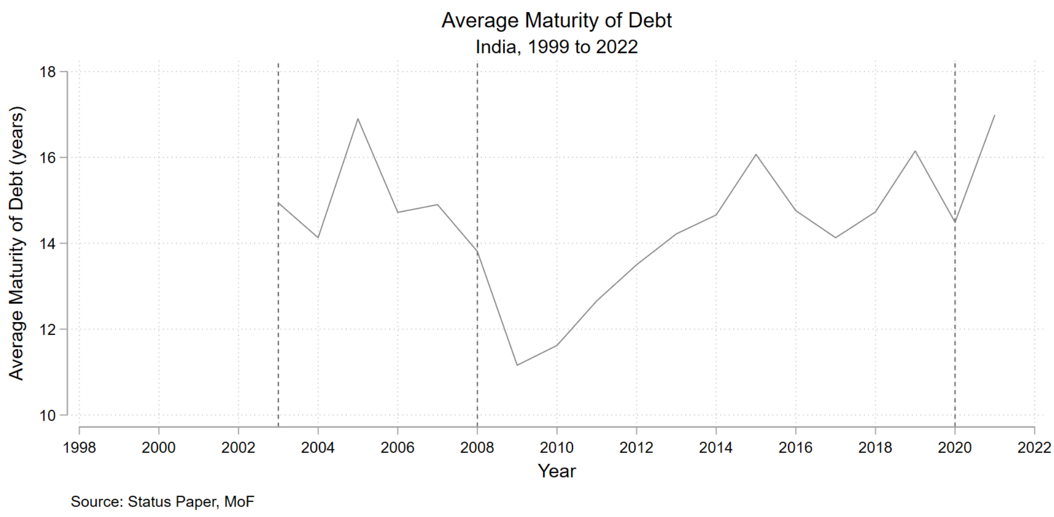

Figure 2: Average Maturity of Debt from the Status Paper of Government Debt

2 Related Literature

Several studies have examined the interactions between monetary and fiscal policies (Leeper (1991), Cochrane (2001), Sims (2011), Bhattarai et al. (2014),Leeper and Leith (2016), Cochrane (2018), Bianchi and Melosi (2017), Bianchi and Ilut (2017))[7]. A central finding in this body of literature is that conventional responses to shocks, such as reducing interest rates to counter demand contractions, lose efficacy when monetary policy becomes subservient to fiscal policy. This is because shocks transmitted through the government's budget can impact inflation. Works by Bianchi and Melosi (2014), Leeper and Leith (2016), Leeper and Li (2017), and Leeper and Zhou (2021) have attributed a significant role to inflation in the realm of optimal monetary-fiscal policy. Recent contributions by Cochrane (2018), Cochrane (2022c), and Cochrane (2022a) study how expectations regarding future primary surpluses and deficits can induce changes in the present real value of the debt-GDP ratio. They assign considerable importance to anticipated and prolonged shifts in price levels, as opposed to abrupt price changes to meet the government's budget constraint.

However, a significant portion of this literature has predominantly concentrated on aggregate debt levels. Given that the aggregate debt is itself made up of securities of varying maturities, the maturity structure of debt is another channel through which the government can act and meet its budget constraint. Our paper also contributes to the literature

[7] A large strand of literature focuses on the aspects of monetary and fiscal policy on the sovereign debt dynamics. Leeper and Plante (2010) analyzes the effects of simple monetary-fiscal policy rules during a credit crunch, emphasizing the importance of policy coordination. Cochrane (2019) explores the relationship between interest rates and fiscal sustainability, highlighting the implications for monetary-fiscal policy coordination while Schmitt-Grohé and Uribe (2017) investigates whether optimal capital-control policy should be counter-cyclical in open-economy models with collateral constraints.

on how fiscal or monetary dominance is transpired. In a fiscally dominant regime, inflation has conventionally been seen as the primary means to maintain sustainable debt levels. Additionally, financial repression has been explored as another channel in this context (Kumhof et al. (2010), De Resende (2007), De Resende and Rebei (2008)). We propose another channel, which is complementary to the other channels studied in the literature, that materializes through the maturity structure.

Building on Das and Ghate (2022), we employ the framework of Hall and Sargent (2011) to analyze our granular-level dataset to explore the potential impact of fiscal dominance on the maturity structure of debt. Notably, our analysis centers explicitly on the market value of debt, as opposed to the face value, to compute the average debt maturity. This approach, as we will demonstrate, differs from the figures presented by the government in its official reports. While Cochrane (2022c) also undertakes a decomposition at the security level to assess the role of debt's maturity structure in the US context, his analysis employs the face value of debt and does not capture explicitly the role of monetary fiscal coordination.

Our work is also related to the work of Buiter and Patel (1992) and Rangarajan and Srivastava (2003) also undertake a debt decomposition using aggregate Indian government debt data. Rangarajan and Srivastava (2003), however, do not cover the inflation targeting period and neither they take into account security level analysis. Moharir (2022) conducts an accounting decomposition for India using aggregate debt data for the time period 1981 to 2017 but he considers aggregate-level debt and not security-level data. However, none of these studies explicitly try to see the role of fiscal-monetary interactions in the evolution of the debt-GDP ratio in India. Moreover, ours is the first study to employ the Hall and Sargent (2011) methodology in examining the influence of fiscal dominance or monetary dominance on the maturity structure of debt.

3 Stylized facts

Figure 3 displays the maturity profile of outstanding central government securities as a percentage of the total outstanding securities. The key takeaway from the figure is the steady increase in long-term securities (maturities exceeding 20 years) as a percentage of the total outstanding securities over the last 2 decades. Additionally, short-term debt (maturities less than 1 year) constitutes less than 10% of the total outstanding debt. This figure underscores the government's commitment to extending the maturity structure of its debt. One noteworthy observation from this graph is the U-shaped curve for securities with maturities between 10 and 20 years, which experienced a sharp increase following the global financial crisis. In India, the 10-year government security is the most actively traded, leading to a substantial liquidity premium, making it a cost-effective choice for the government to issue more to meet the demand.

Figure 4 illustrates the maturity periods of securities auctioned between 1995 and 2022. Each dot on the graph represents either the auction of issuance of a new security or its reissuance, with the date on the x-axis and maturity on the y-axis. Notably, a distinct pattern emerges resembling a saw-tooth shape [8]. This pattern reflects that sequential s in a given vintage have approximately the same maturity and indicates that securities initially auctioned with a maturity period of j years are subsequently repurchased and reissued with a $j - \epsilon$ maturity. Since 2006, the frequency of longer-term securities auctions has increased, with some securities having maturity periods of up to 40 years, marking the longest maturity. Over time, there is a rising trend in the proportion of securities with maturities exceeding 10 years being auctioned.

We weight the maturity period of each auction in Figure 4 with the auctioned amount (denoted in Rs. billion). One notable feature is the significant presence of large auctioned amounts in the higher maturity bucket (see the bunching of red and green dots in the figure). This shows that not only is the government reissuing more debt with a longer maturity (residual), but it is also doing so for larger auctioned amounts, indicating the extent of borrowing undertaken by the government with a longer maturity period.

To reconcile the fact that the government issues a wide range of securities with different maturities with a preference towards the longer maturities, Appendix Figure 15 plots the weighted average yield of the central government securities along with the upper and lower bounds around each point for each year. It is quite evident from the figure about the wide array of securities offered by the government because of the range of yields (presented by the length of the bars). Notice that the weighted average is always skewed towards the upper bound of yield, revealing the larger share of securities with longer maturities which demand higher yields since in India the yield curve has always remain positively sloped[9].

Another notable feature from Figure 4 is the increase in the auction of new securities in the highest maturity bracket of 30-40 years. One of the reasons for this pattern is the endeavor of the DMO to minimize the rollover risk associated with increased borrowings to support rising spending needs due to the COVID-19 pandemic. The auction data shows that the overall average maturity of debt has been rising for the past 16 years. Also, as per the Status Paper on Government Debt (2020-2021 p. (iii)) the debt management strategy of

[8] Bigio et al. (2023) show the same pattern exists for Spain using auction-level granular level data ,and argue that this pattern is quite common in the other bond markets where the government tries to take advantage of the continuous issuances in several maturities and not bunching them in just a single maturity.

[9] See this RBI discussion paper for more details

the government is as follows:[10]

"It has been the endeavour of the Government to elongate the maturity profile of its debt portfolio with a view to reduce the roll-over risk."

which clearly spells out this maturity lengthening strategy to be followed by the DMO on behalf of the government. This aspect bears significance due to the reasons outlined in the status paper, which encompass objectives such as minimizing rollover risk and broadening the investor base to include pension funds and insurance funds.

Our analysis of security-level data reveals a rising trend in the average maturity of debt. However, the calculated maturity level is somewhat lower compared to the figures indicated in the status paper (Figure 5). This discrepancy can be attributed to the methodology employed for computing average maturity. While the status paper adopts a straightforward weighted average approach based on the number of securities issued with varying maturities, our approach factors in the market value of the securities. For instance, if the government issues two securities – one with a 5-year term and the other with a 10-year term – the status paper's average maturity would be 7.5 years. In contrast, our method considers the relative market values of these securities, which might lead to different outcomes. The market might value for the 10-year security is higher due to its higher liquidity, thus yielding divergent estimates in comparison to the simple weighted average used by the status paper.

While examining the evolution and composition of the supplied securities is essential, an equally significant aspect is analyzing the debt holders. Figure 6, illustrates the ownership pattern of central government debt using monthly data from RBI, spanning from 2007 to 2022. Notably, commercial banks have historically been the principal holders of these securities; however, their share has dwindled over the past six years. In contrast, the shares of insurance companies and pension funds have been on the rise[11]. This shift bears weight as insurance companies and pension funds exhibit a preference for longer maturity bonds, whereas commercial banks tend to favor short-term securities. With the elongation of the debt maturity structure, insurance companies are poised to become the primary investors in these securities. Consequently, the implications of a lengthened debt maturity structure will have a substantial impact on these debt holders. Nonetheless, it's important to recognize that commercial banks and other institutional investors still possess a significant portion of these securities, underscoring the tight nature of the bond market. As a result, any repercussions stemming from a lack of coordination between the monetary and fiscal authorities will disproportionately affect these institutional investors.

[10] Status Paper on Government Debt

[11] Though RBI permits retail investors to invest in these securities, their share remains minimal. This pattern is consistent when considering the public holding of these securities.

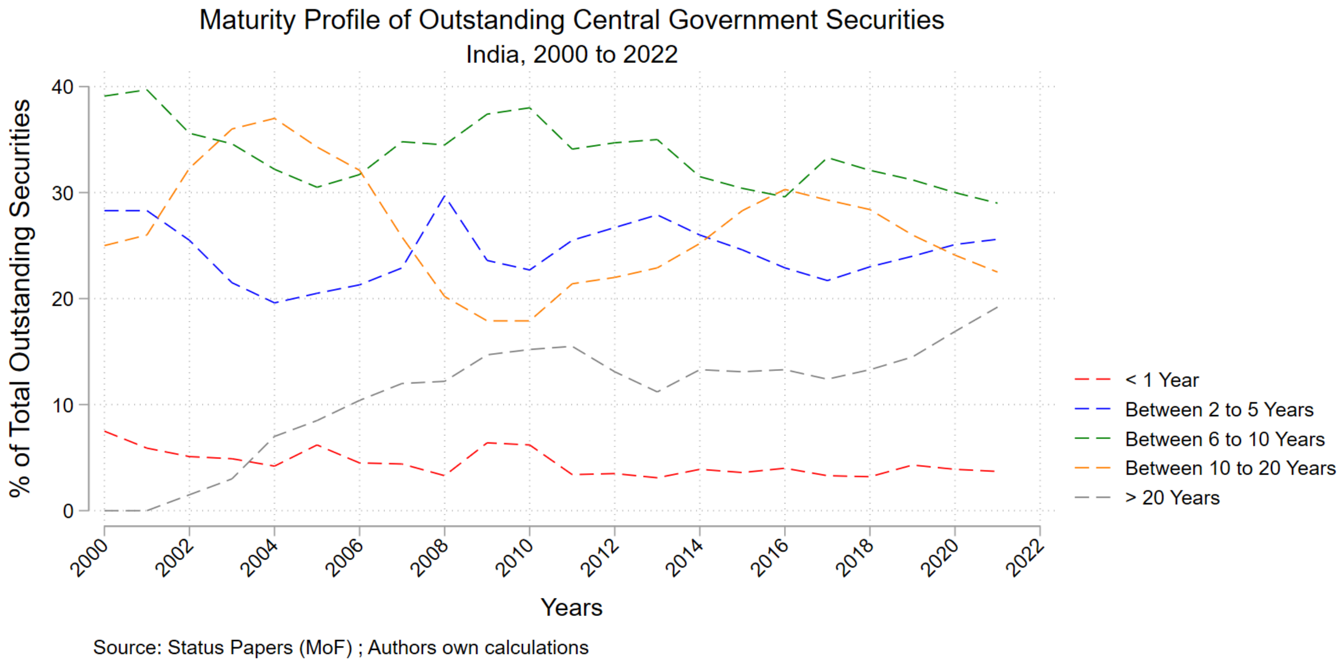

Figure 3: Maturity profile of outstanding securities as % of total outstanding

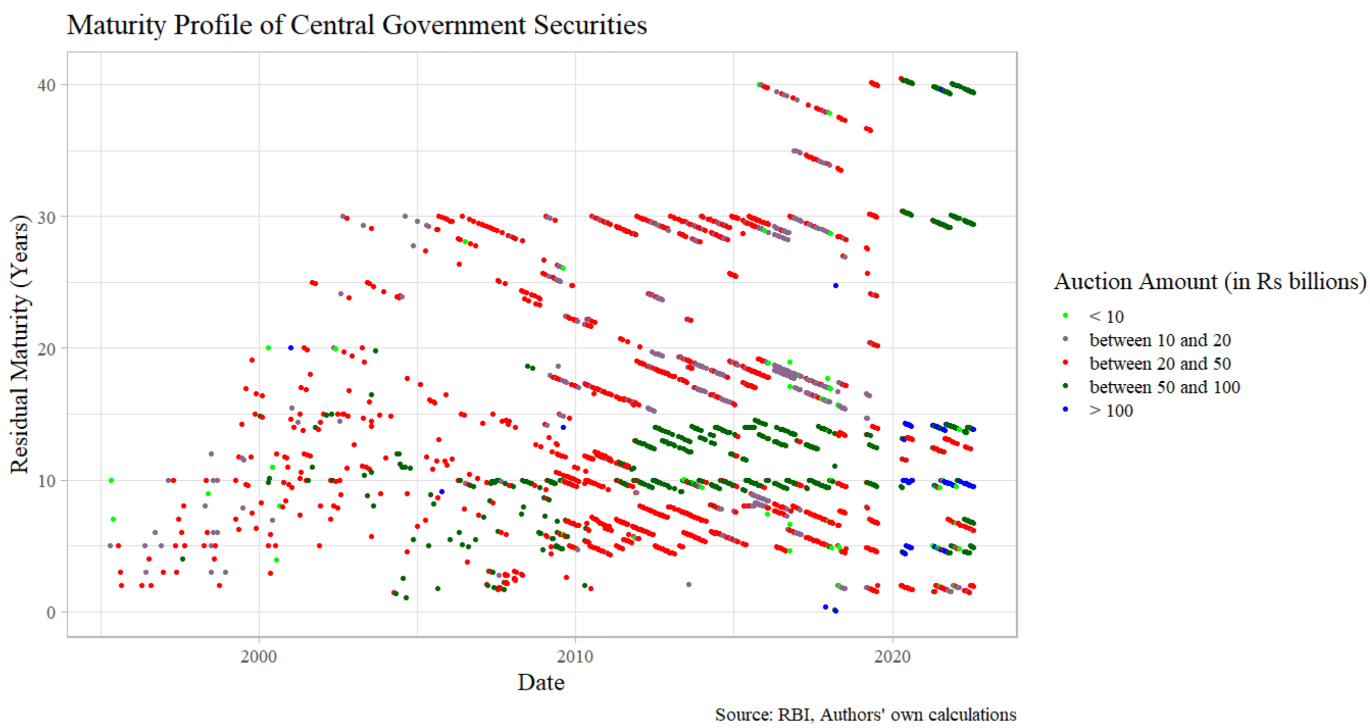

Figure 4: Auction of new issuances and reissuances of Central Government Dated Securities

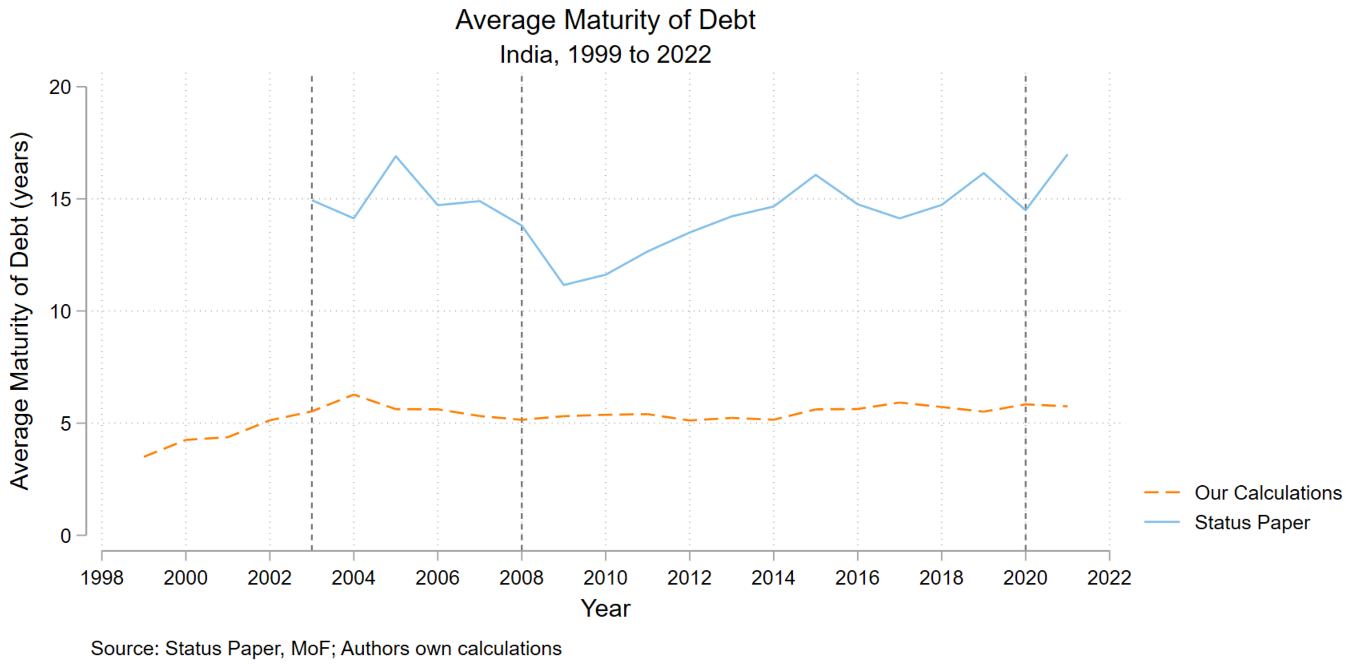

Figure 5: Average Maturity of Debt

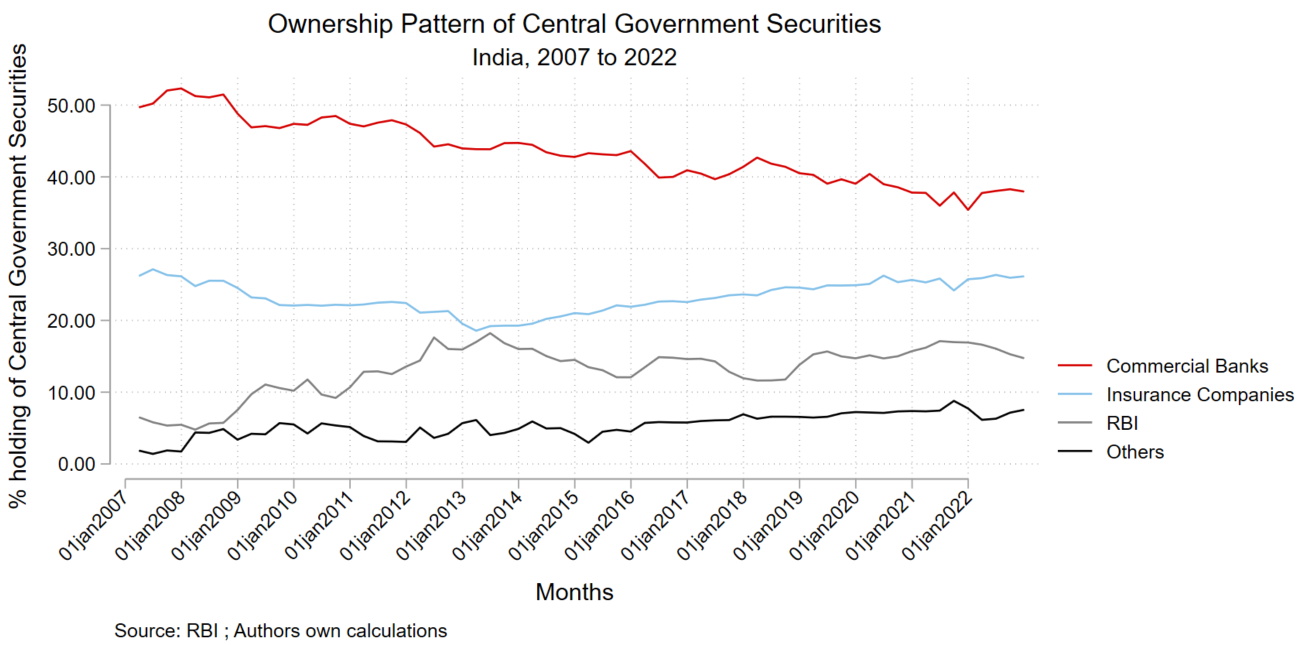

Figure 6: Ownership Pattern of Central Government Debt

4 A Simple Model

We construct a static model to analyze the optimal maturity structure determined by the Debt Management Office (DMO), which faces a trade-off between issuing long-term and short-term debt. Our model highlights the potential implications of the optimal maturity structure under different regimes, specifically, a fiscally dominant regime or a monetary dominant regime. First, we describe the model environment, emphasizing its key aspects and outlining the problem faced by the DMO. Subsequently, we illustrate how fiscal dominance impacts the debt's maturity structure and conduct comparative statics to dissect the contributions of the short-term interest rate, monetary-fiscal coordination, and the rollover risk associated with the debt level in influencing the average debt maturity.

4.1 Model Environment

Our model builds on the work by Greenwood et al. (2015), and He and Xiong (2012), where we look at the problem of a Debt Management Office (DMO) which wants to minimize interest costs for the government in raising additional debt while fetching the desired amount to be raised. Diverging from Greenwood et al. (2015), our model explicitly examines the role undertaken by the DMO—a distinct entity from the government—yet diligently committed to reducing costs for the government. Moreover, our study acknowledges the prevalent structure in various emerging market economies, including India, where the DMO is a department within the monetary authority (Central Bank).

We consider a simple static model in order to study the effect of how different regimes play a role in determining the maturity structure of debt. We also abstract away from any default risk and foreign currency debt[12]. The main objective of the DMO is therefore to achieve the lowest cost of financing and spreading the maturity of its borrowings across the yield curve in order to reduce the risks emanating from holding either too much long-term debt or too much short-term debt. As we will see, this trade off between short-term and long-term debt will be important in determining the ability of our model to trace out the empirical results.

For simplicity, the DMO can issue only two types of bonds: short- term and long-term[13]. However, there are potential trade-offs associated with each of them. Short-term bonds facilitate the immediate fulfillment of financial requirements, as they are evaluated by investors at a comparably lower cost, given that short-term rates tend to be lower than those of long-term bonds (reflected by an upward-sloping yield curve). However, conversely,

[12] This is a safe assumption, since in India the percentage of external debt is $\approx 4\%$

[13] Even with different types of bonds with varying degrees of maturities, the underlying results will not change

abrupt shifts in interest rates could lead to price volatility for these bonds, subsequently influencing their market value and potentially introducing refinancing risks when the short-term debt matures for payment. Long-term bonds, on the other hand, do not suffer from rollover/refinancing risks due to sudden changes in the interest rates but investors demand excess returns or term-premium for holding long-term bonds, which increases the cost for raising debt for the government (Greenwood and Vayanos (2014), Greenwood et al. (2015)). The DMO tries to balance this tradeoff by choosing the optimal maturity structure[14]. Given this backdrop, we now formalize the problem of the DMO.

Consider a DMO faced with an initial accumulated Debt-GDP ratio $D > 0$ at time t [15]. Like Greenwood et al. (2015), we assume that the government can finance itself by either issuing bonds or by levying taxes. Since the DMO needs to take care of the total amount of debt, so as to keep debt at a sustainable level, it takes into account the fact that a higher level of spending will lead to even higher borrowings (in order to pay off the accumulated debt) which increases the tax burden. Therefore, while setting the optimization problem, the DMO takes into account the variance of future spending needs of the government as well as the volatility of taxes as higher taxes will reduce the need to increase borrowings, but will entail a deadweight cost on taxpayers. The burden will be shifted from bondholders to the taxpayers [16].

For simplicity assume that the DMO can issue only 2 kinds of bonds: short-term and long-term. Let α be the fraction of debt that is short-term which means that (1-α) proportion of debt is long-term. Let r be the rate of interest on the short-term debt and $\hat{r}$ be the rate of interest on the long-term debt with $\hat{r} > r$. Let τ be the tax revenue to GDP ratio and G be the government's primary deficit to GDP ratio (which denotes the extra spending). Then the financing equation, which is different from the budget constraint, for the government is given by

$$\underbrace{\tau}_{receipts} = \underbrace{\alpha r D + (1-\alpha)\hat{r} D + G - \pi \alpha D}_{expenditure} \tag{1}$$

where the left hand side shows the receipts of the government. The first two terms on the right hand side are the interest expenditure. The last term on the right hand side denotes the fact that inflation reduces the real Debt-GDP ratio and therefore leads to a reduction of the same.

[14]We assume that the DMO only decides the maturity structure, since in India along with many other countries, the securities are auctioned as per yield based or price based and hence market forces determine the prices of the securities. Therefore, the DMO only caters to the design of the security and not its prices.

[15]Since we consider a static model, we refrain away from putting the time subscript for the variables.

[16]Note that the DMO is not bothered by the burden of level of taxes in the economy but because higher volatility of taxes will have implications on borrowings, the DMO consider this in its optimization problem. Moreover, given that the DMO in most of the EMEs including India resides in the Central Bank of the country, it need to take into account the social cost originating from levying higher taxes on the economy.

We assume that short-term rates vary in the future while long-term rates depend on the expected future short-term rate (Cochrane and Piazzesi (2005))[17]. As discussed earlier, issuing a larger amount of short-term debt not only increases refinancing risks but also amplifies the necessity to generate additional tax revenue, thus increasing tax volatility. Following Bohn (1990), we assume that taxation entails deadweight cost given by

$$\frac{\lambda}{2} Var(\tau) \tag{2}$$

where λ is a positive parameter. The maturity structure plays an important role in determining the extent of rollover risk the government faces, since a higher proportion of short-term debt will lead to more rollover risks (He and Xiong (2012)). Assume that the real market value of new bonds issued is denoted by $\mathcal{P}(r, \alpha, \pi)$, where market value depends on the short rate (r), proportion of short-term debt (α), and inflation (π) with

$$\mathcal{P}_\alpha(r, \alpha, \pi) > 0\,; \mathcal{P}_r(r, \alpha, \pi) < 0\,; \mathcal{P}_\pi(r, \alpha, \pi) < 0$$

The first term shows that new short-term bonds are valued more by investors (He and Xiong (2012)), market value falls with an increase in r (inverse relation between price and yield)[18], and inflation reduces the real market value of debt. Further assume that be the proportion of principal payment that is due in a particular time period. The total principal payment is given by $p\alpha D$. There will be rollover risk if

$$p\alpha D - \mathcal{P}(r, \alpha, \pi) > 0 \tag{3}$$

which implies that the amount to be paid on principal repayment is greater than the market value of t new bonds issued.

The DMO's Optimization Problem

We can write the optimization problem for the DMO as

$$\min_{\alpha} \left[\underbrace{(\alpha r D + (1-\alpha)\hat{r} D)}_{\text{Interest Costs}} + \underbrace{\frac{\lambda}{2} Var(\tau)}_{\text{deadweight cost of taxation}} + \underbrace{(pD\alpha - (\mathcal{P}(r, \alpha, \pi))}_{\text{Rollover risk}} \right] \tag{4}$$

[17] Ang and Piazzesi (2003) presents a no-arbitrage vector autoregression model to examine the term structure dynamics, including the impact of expected future short-term rates, while Gurkaynak et al. (2007) analyzes the U.S. Treasury yield curve and discusses how market expectations of future short-term rates influence long-term interest rates. For more details see Bauer and Rudebusch (2016),and Joslin et al. (2019)

[18] See Greenwood and Hanson (2013), Haddad and Sraer (2018), and Cieslak et al. (2019) for more details.

substituting the value of $Var(\tau)$ from equation 1, we have

$$\min_{\alpha}\Bigg[(\alpha r D+(1-\alpha)\hat{r}D)+\frac{\lambda}{2}\big[D^2Var(\pi)(\alpha)^2+D^2Var(r)(\alpha)^2+ \\ Var(G)+2DCov(G,r)(\alpha)- \\ 2DCov(G,\pi)(\alpha)-2D^2Cov(\pi,r)(\alpha)^2\big] \\ -\left[\mathcal{P}(r,\alpha,\pi)-pD\alpha\right]\Bigg] \tag{5}$$

The DMO chooses the proportion of short-term debt, which given the assumption of only 2 types of bonds, also gives us the long-term bonds proportion.The FOC with respect to α gives

$$\alpha^*=\underbrace{\frac{(\hat{r}-r)}{\lambda DA}}_{\text{spread effect}}+\underbrace{\frac{(\mathcal{P}_\alpha(r,\alpha,\pi)-pD)}{\lambda D^2 A}}_{\text{revaluation effect}}-\underbrace{\frac{Cov(r,G)}{DA}}_{\text{regime effect}}+\underbrace{\frac{Cov(G,\pi)}{DA}}_{\text{debt deflation effect}} \tag{6}$$

with $A>0$

Equation 6 shows that the optimal α depends on four different effects. The *spread effect* shows that higher the interest rate on long-term bonds over the short-term bond, higher will be α, meaning that higher will be the proportion of short-term debt. The second term, *revaluation effect*, shows that if the new bonds are valuable over and above the amount due in principal payment, the DMO will issue more of short-term debt (α goes up) since investors value short-term debt more. The next term, which denotes the *regime effect*, shows the role of the monetary and fiscal co-ordination in determining optimal α, while the last term, *debt-deflation effect*, shows that for a given covariance between government spending and inflation, a higher debt-GDP ratio will lead to more long-term debt issued.

After determining the optimal value of α concerning various effects, we now turn our attention to analyzing how fiscal dominance can influence the choice of the optimal maturity structure. We will demonstrate that as the debt level rises, the Debt Management Office (DMO) may choose to elongate the maturity structure. This strategy is prudent, especially when considering the rollover risk. Interestingly, even in the absence of rollover risk, a rising debt level can still prompt the DMO to extend the maturity structure. This behavior is more likely to occur in a fiscally dominant regime. Additionally, we will explore how changes in the policy rate (equivalent to the short-term rate in our model) can affect the average maturity structure, highlighting the impact of fiscal dominance on this relationship.

4.2 Impact of Fiscal Dominance on Maturity Structure of Debt

Fiscal Dominance is a special form of monetary-fiscal interaction where the monetary authority acts "passively" while the fiscal authority is the first mover or acts "actively" (Leeper (1991)). In a scenario of fiscal dominance, fiscal policy establishes primary surpluses autonomously from actual government debt, coupled with a monetary policy aimed at preventing interest payments on existing debt from causing rapid debt escalation. Under this framework, inflation is influenced by fiscal policy, while monetary policy takes on a passive role in maintaining debt stability. The literature has mainly focused on this aspect of fiscal dominance where it is inflation in the current period will play the principal role in keeping the real value of debt at a sustainable level(Kumhof et al. (2010), Cochrane (2022c)). While the maturity structure of debt has always been a background consideration, it has not received explicit attention in the literature. In our model, we establish a connection between the maturity structure of debt and fiscal dominance by examining the levels of debt and interest rates, two critical factors for the Debt Management Office (DMO).

In our theoretical model, we assume the debt level is given to the DMO. However, we demonstrate that as the debt level continues to rise, the DMO may opt to lengthen the maturity structure. This strategy is a prudent response to mitigate risks associated with rolling over debt. Such a strategy is understandable and is commonly pursued by DMOs when confronted with these risks.

However, we show that even when the debt is at a level when the rollover risk is not binding, the DMO may still choose to extend the maturity structure. Interestingly, this phenomenon occurs under both types of regimes, but it is more likely to manifest under a fiscally dominant regime. Why is this the case? When debt levels are on the rise, regardless of the absolute debt level, a fiscally dominant regime prompts the DMO to extend the maturity structure. This strategic move effectively shifts the burden of debt repayment into the future, providing vital support for the sovereign government's spending decisions. It enables the government to pursue budgetary deficits actively without immediate concerns about repayment[19].

[19] Another way to look at is by looking at the debt valuation equation, which equates the total market value of debt at the current period with the expected present value of primary surpluses in the future. This equation needs to be satisfied every period in order to keep debt at a sustainable level and is actually an equilibrium condition (see Cochrane (2022c) for more details).

$$\frac{\sum_{j=0}^{\infty} q_t^j B_t^j}{P_t} = \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{7}$$

where B_t^j is the the nominal value of bond at time t of maturity j, q_t^j is the price of such a bond, P_t is the price-level in the economy, s_t denotes primary surplus, and β is the discount factor. When the government increases its in time period t, then the left hand side of the above equation falls. In order to hold equality, Another channel through which fiscal dominance can influence the maturity structure of debt is via the short-term interest rate, which is equivalent to the policy interest rate controlled by the central bank in our model. We posit that under a fiscally dominant regime, we would observe a negative relationship between the short-term interest rate and the maturity structure of debt. This may appear counter intuitive, as a decrease in the short-term interest rate would logically reduce interest payments on short-term debt and, therefore, should lead to an increase in short-term bond issuances. However, the rationale behind this behavior is compelling.

When the DMO issues long-term bonds, it effectively 'locks in' the prevailing lower interest rate on these bonds, even if they mature in the future. This strategy becomes even more appealing if the DMO anticipates future interest rate increases. The expectation of rising future interest rates is driven by the central bank's response to expected future price increases, which aims to maintain the real value of debt at a sustainable level [20]. Thus, by increasing the maturity structure when short-term interest rates are lower, it not only reduces the interest payments for the government but also take advantage of the longer repayment period for these bonds.

To gauge the impact of fiscal dominance on the average maturity structure through the two different channels mentioned above, we perform some comparative statics to quantify these effects. Specifically, we analyze the effect of increasing the debt-GDP ratio on the optimal average maturity level chosen by the the DMO. We found that increasing debt-GDP can lead to a longer maturity structure even under the condition of no rollover risk and this effect is more likely to be generated under a fiscally dominant regime, which we also proposed in our first channel mentioned above. To verify our conjuncture proposed in the second channel, we also perform the comparative statics between the short term interest rate and the optimal maturity structure.

The results from this exercise show that fiscal dominance leads to shortening of debt when the interest rate is going down, supporting the hypothesis that we mentioned above. But before we proceed to the next section to perform these comparative statics, we must de-

the left-hand side needs to fall. By supplying more of long-term debt, the price of such bonds fall and that lowers the numerator of the left-hand side and therefore, even without a sudden increase in the price level this period.

[20] This argument builds upon the previous footnote. When the DMO extends the maturity structure, it capitalizes on the advantage of lower interest costs. In this scenario, the central bank, playing a 'passive' role mandated to maintain debt at a sustainable level, allows price levels to rise in the future (at the time when the long-term bonds come due for repayment). Consequently, the debt-valuation equation remains satisfied in the future period. By locking in the current lower interest rates, the DMO achieves a twofold benefit: it not only reduces current *nominal* interest payments but also curtails future *real* interest payments for the government with this long-term debt.

fine fiscal and monetary dominance within the context of our model. To do that, the sign of the last two terms on the right-hand side of equation 6 becomes important. Under a fiscally

	$Cov(G,r)$	$Cov(G,\pi)$	ϕ (Taylor Principle)
Fiscal Dominance Regime	<0	>0	$\in (0,1)$
Monetary Dominance Regime	>0	>0	>1

Table 1: Different Regimes

dominant regime, as mentioned above, the central bank cuts the interest rate when there is an increase in government spending ($Cov(G,r) < 0$) while the opposite is true under a monetary dominance regime. We assume that all government spending is inflationary, irrespective of the regime, and thus $Cov(G,\pi) > 0$. The last column shows the magnitude of the Taylor rule coefficient under the two regimes. The next two sections provide the results of the comparative statics.

4.3 Comparative Statics 1

Our first comparative statics tries to explain the relation between the average maturity of debt and the debt-GDP ratio. Similar to findings in studies such as Greenwood and Vayanos (2014) and Cochrane (2022a), we, too, establish a positive correlation between these factors, as evidenced in Figure 12. However, we rigorously examine this relationship within our model, particularly emphasizing its potential manifestation under conditions of fiscal dominance. With an increase in debt-GDP ratio, the prudent approach for the Debt Management Office (DMO) is to elevate the maturity structure of debt, effectively mitigating rollover risk. Remarkably, our analysis unveils an additional insight: even in the absence of imminent rollover risk, the DMO can strategically bolster the maturity structure, a phenomenon attributed to fiscal dominance. This phenomenon arises from the fact that elevated issuance of long-term securities empowers the DMO, a branch within the Central Bank, to factor in the government's capacity to elevate debt levels through augmented expenditures. This strategy renders the debt sustainable in the present, as future repayments are deferred, ensuring fiscal sustainability for the current period.

In order to capture this fact, we compute the sign of $\frac{\partial \alpha^*}{\partial D}$. Our prior, what we get from our empirical analysis, is that the sign of

$$\alpha^* = \frac{(\hat{r} - r)}{\lambda D A} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D^2 A} - \frac{Cov(r,G)}{DA} + \frac{Cov(G,\pi)}{DA}$$

$\frac{\partial \alpha^*}{\partial D} < 0$ is satisfied when

$$Cov(r, G) < \frac{(\hat{r} - r)}{\lambda} + Cov(G, \pi) + \frac{(2\mathcal{P}_\alpha(r, \alpha, \pi) - pD)}{(\lambda D)}$$

We need to check under which regime and given the signs of the other terms, when the above inequality gets satisfied. Note the importance of rollover risk, which is given by the last term $(2\mathcal{P}_\alpha(r, \alpha, \pi) - pD)$, in determining whether this inequality will get satisfied or not. This leads us to our first proposition.

Proposition 1

Under the assumption of positive accumulated debt-GDP of $D > 0$ and $Cov(G, \pi) > 0$, $\frac{\partial \alpha^*}{\partial D} < 0$ can occur in

- a **fiscally dominant regime**

$$\frac{\partial \alpha^*}{\partial D} < 0 \quad if \begin{cases} D < \frac{2\mathcal{P}_\alpha}{p} & \text{with no rollover risk} \\ \frac{(\hat{r} - r)}{\lambda} + Cov(G, \pi) > \frac{(pD - 2\mathcal{P}_\alpha)}{\lambda D} & \text{with rollover risk} \end{cases}$$

- a **monetary dominant regime**

$\frac{\partial \alpha^*}{\partial D} < 0$ if

$$\underbrace{Cov(r, G)}_{>0,\text{under MD}} < \underbrace{\frac{(\hat{r} - r)}{\lambda}}_{>0,\text{excess returns}} + \underbrace{Cov(G, \pi)}_{>0} + \underbrace{\frac{(2\mathcal{P}_\alpha(r, \alpha, \pi) - pD)}{(\lambda D)}}_{>0 or <0} \tag{8}$$

The above proposition says that $\frac{\partial \alpha^*}{\partial D} < 0$ can happen under a fiscally dominant regime, if the debt-GDP ratio is within some threshold (so that there is no rollover risk), and even when the threshold crosses and the debt-GDP increases with an increase in the possibility of rollover risk, the DMO lengthens the maturity. However, this trend in maturity extension reverses once the debt-GDP ratio exceeds a higher threshold delineated by $\frac{2\mathcal{P}_\alpha}{p - h}$, as depicted in Figure 7. Beyond this second threshold, the debt-GDP ratio becomes so elevated that long-term bonds lose favor among investors. Consequently, to further elevate the debt level, the DMO must adopt a strategy of decreasing the maturity structure, given that short-term bonds hold greater appeal to investors.

Under monetary dominance, we get a similar picture as shown by Figure 8. The negative relation between average maturity and the debt-GDP ratio can still persist during the no rollover risk regime but now along with debt being lower than the same threshold as under he case of fiscal dominance, an additional constraint given by $\frac{(\hat{r}-r)}{\lambda} + Cov(G,\pi) > Cov(r,G)$ is required to be satisfied in order to generate the same result. Thus the result under monetary dominance is restrictive in that sense. A noteworthy aspect is the threshold debt level beyond which the relationship between average maturity and the debt-GDP ratio reverses. As depicted in Figure 8, this threshold lies to the left of the threshold observed under fiscal dominance. This signifies a lower debt-GDP ratio. This distinction arises because, despite the rationale for extending the maturity structure with an increasing debt-GDP, the monetary authority's "active" role curbs the debt-GDP ratio, placing this threshold below that seen in a fiscal dominance regime. This means that the level of debt-GDP is higher in the fiscally dominant regime as compared to the monetary dominant regime depicting the increase in debt that the central bank (through its DMO) allow when it plays a "passive" role

Therefore, our findings reveal that considering the signs of different terms and the specific regime under consideration, the inverse connection between the average maturity of debt and the debt-GDP ratio is more inclined to be evident under a *fiscally dominant* regime. This outcome underscores that even in the absence of rollover risk, the Debt Management Office (DMO) can protract the maturity of debt. This strategic maneuver enables the government to enhance its fiscal capacity by accommodating increased spending requirements while postponing the repayments to future periods.

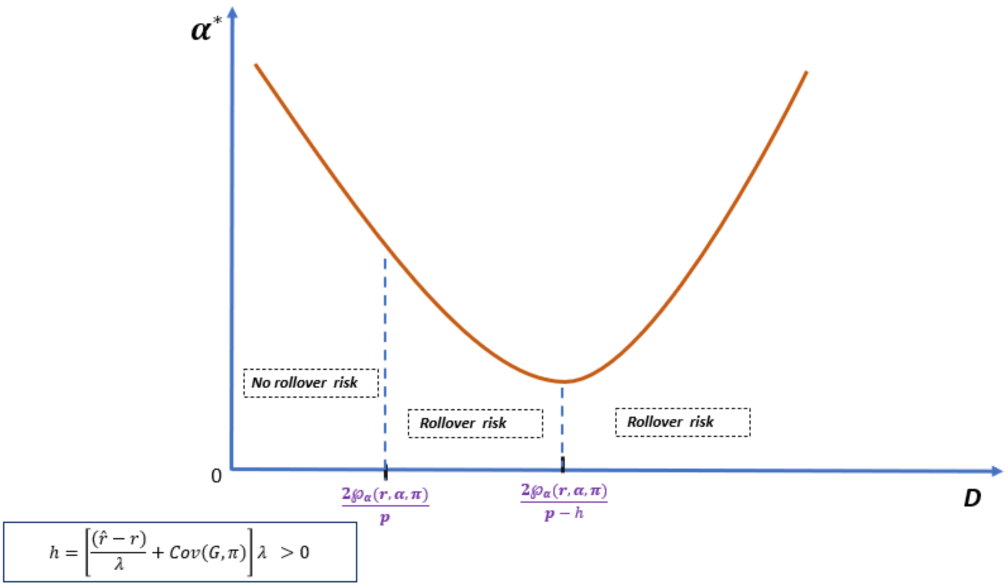

Figure 7: $\frac{\partial \alpha^*}{\partial D}$ under Fiscal Dominance

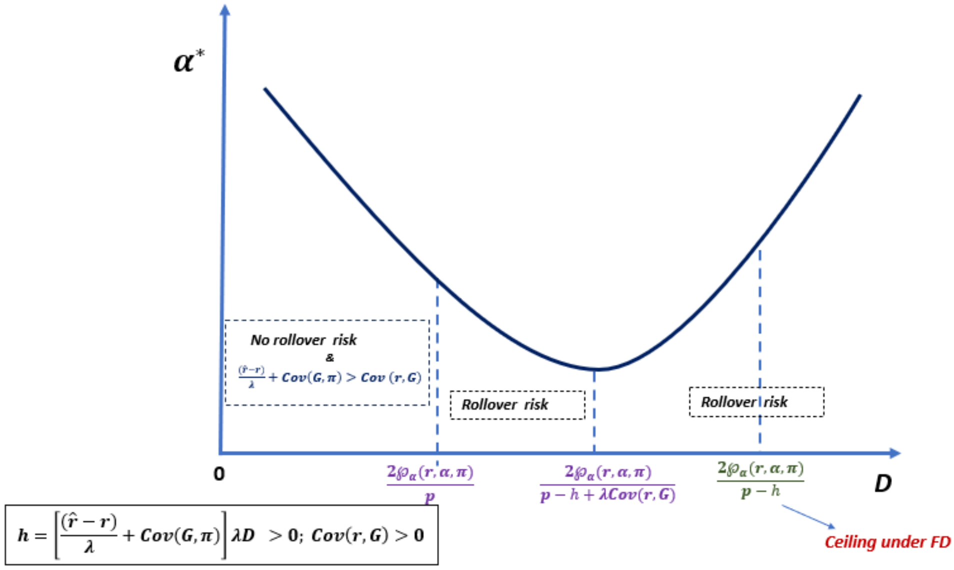

Figure 8: $\frac{\partial \alpha^*}{\partial D}$ under Monetary Dominance

4.4 Comparative Statics 2

Next, we examine the correlation between the maturity structure of debt and the short rate, commonly regarded as the risk-free rate. As the short rate escalates, interest obligations on short-term bonds surge due to the fixed interest rates on long-term bonds. In response to this, the Debt Management Office (DMO) should theoretically reduce the issuance of short-term debt to mitigate the interest payment burden.

However, as proposed earlier, the second channel of fiscal dominance will imply that the average maturity will be higher when the short-rate falls, in order to "lock-in" the current lower interest rates, expecting that the future short-term rates will be higher. This is because in a fiscally dominant regime, the central bank will be persuaded to use a gradual increase in prices (future inflation) in order to maintain the real value of debt. Thus, the inflation-targeting central bank will increase rates when inflation in the future rises but as the fixed-rate long term bond already has lower rates locked-in earlier, the interest payment burden does not goes up for debt issued in the past. Our model shows that the intricate relationship between maturity and short-rate is contingent upon the specific regime and, notably, the extent of rollover risk present.

We want to ascertain the sign of $\frac{\partial \alpha^*}{\partial r}$, for which we start again with 6 that gives α^* as

$$\begin{aligned}\alpha^* =& \frac{(\hat{r}-r)}{\lambda D \mathrm{A}} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D^2 \mathrm{A}} - \frac{Cov(r,G)}{D\mathrm{A}} + \frac{Cov(G,\pi)}{D\mathrm{A}} \\ =& \frac{1}{D\mathrm{A}}\left[\frac{(\hat{r}-r)}{\lambda} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right]\end{aligned} \tag{9}$$

where

$$\mathrm{A} = Var(r) + Var(\pi) - 2Cov(\pi, r) \implies \mathrm{A} = Var(r-\pi) = Var(r_{real}) > 0$$

It can be shown that $\frac{\partial \alpha^*}{\partial r} > 0 \implies$

$$\begin{aligned}&\mathrm{A}\left[\frac{-1}{\lambda} + \frac{\mathcal{P}_{\alpha r}(r,\alpha,\pi)}{\lambda D} - \frac{\partial Cov(r,G)}{\partial r}\right] \\ &> \frac{\partial \mathrm{A}}{\partial r}\left[\frac{(\hat{r}-r)}{\lambda} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right]\end{aligned} \tag{10}$$

where as like the previous comparative statics exercise, the sign of the individual terms determine under what condition the above inequality holds. Now, $\mathcal{P}_{\alpha r}(r,\alpha,\pi)$ is the change in market value when the interest rate changes and by assumption, $\mathcal{P}_{\alpha r}(r,\alpha,\pi) < 0$. However, the sign of two the other terms before: $\frac{\partial \mathrm{A}}{\partial r}$ and $\frac{\partial Cov(r,G)}{\partial r}$ need to be ascertained. In order to see the sign of these two terms, we need to make some assumption regarding the relationship between r and G, and how A is related to r.

We assume that the central bank follows a simple Taylor rule to set the short-rate (which we assume to be equivalent to the policy rate) given as

$$r = \phi(\pi - \pi^*)$$

where $\phi \gtrless 1$ depending on the regime we are in.
Moreover, if we assume that the variance of the short-rate, r, increases with an increase in r itself, $\frac{\partial Var(r)}{\partial r} > 0$, then it can be shown that

$$\frac{\partial \mathrm{A}}{\partial r} > 0$$

which means that the variance of the real interest rate given by A also increases with an increase in the short-term rate.

In order to ascertain the sign of $\frac{\partial Cov(r,G)}{\partial r}$, we make use of the financing equation of the government given by 1 , and by using the definition of variance we can write

$$\begin{aligned} Var(\tau) =& Var(\alpha r D + (1-\alpha)\hat{r} D + G - \pi \alpha D) \\ &= Var(r) D^2 \alpha^2 + Var(G) + Var(\pi) D^2 \alpha^2 + 2D\alpha Cov(r, G) \\ &- 2D\alpha Cov(\pi, G) - 2D^2\alpha^2 Cov(\pi, r) \end{aligned} \tag{11}$$

which coupled with our assumption of $\frac{\partial Var(r)}{\partial r} > 0$ gives us

$$\frac{\partial Cov(r, G)}{\partial r} < 0$$

This shows that the increase in r leads to a decrease in the strength of the relationship between the covariance of r and G.This says that even though the sign of covariance between r and G can vary depending upon the particular regime yet the magnitude of that relationship falls with an increase in r. Details are in the appendix. Given the sign of these terms along with our assumptions, Proposition 2 summarizes the results.

Proposition 2

Under the assumption of a positive accumulated debt-GDP of $r > 0$ and $\frac{\partial Var(r)}{\partial r} > 0$, $\frac{\partial \alpha^*}{\partial r} > 0$ occurs in

- a **fiscally dominant regime**

$$\frac{\partial \alpha^*}{\partial r} > 0 \quad if \begin{cases} r > \lambda \left[B + C - Cov(r, G) + \frac{\mathcal{P}_\alpha - pD}{\lambda D} \right] & \text{with no rollover risk} \\ r > \lambda \left[B + C - Cov(r, G) - \frac{pD - \mathcal{P}_\alpha}{\lambda D} \right] & \text{with rollover risk} \end{cases}$$

- a **monetary dominant regime**

$$\frac{\partial \alpha^*}{\partial r} > 0 \quad if \begin{cases} r > \lambda \left[B + C - Cov(r, G) + \frac{\mathcal{P}_\alpha - pD}{\lambda D} \right] & \text{with no rollover risk} \\ r > \lambda \left[B + C - Cov(r, G) - \frac{pD - \mathcal{P}_\alpha}{\lambda D} \right] & \text{with rollover risk} \end{cases}$$

where
$B = \frac{\hat{r}}{\lambda} + Cov(G, \pi) + A\left(\frac{\partial A}{\partial r}\right)^{-1} \left[\lambda^{-1} - \frac{\mathcal{P}_{\alpha r}}{\lambda D}\right];$ and $C = A\left(\frac{\partial A}{\partial r}\right)^{-1} \left(\frac{\partial Cov(r, G)}{\partial r}\right)$

Proposition 2 shows that, depending on the presence of rollover risk or its absence, our proposed conjecture that $\frac{\partial \alpha^*}{\partial r} > 0$ can be satisfied under both the fiscally dominant or the monetary dominant regime. Depending on the regime, we get particular thresholds where the negative relationship between the short-term interest rate and the average maturity of debt can be observed. In both the regimes, the thresholds – whether rollover risk is present or not – are notably lower in the monetary dominance regime compared to the fiscal dominance regime. To see this note that, given that the amount od debt-GDP (D) is hold fixed for this comparative static exercise, the term

$$B + C - Cov(r, G)$$

is positive under a *fiscally dominant regime* and therefore the threshold under no rollover risk is at a higher level (as we add a positive revaluation effect to it)than that of the rollover risk regime (since we take out a positive term). The same argument goes under the case of *monetary dominance regime* [21].

Now comparing between the two regimes, we can see that irrespective of the presence or absence of rollover risk, the thresholds are unambiguously higher under a fiscal dominance regime relative to the monetary dominance regime[22]. This means that the DMO lengthen the maturity structure of debt with an increase in the the interest rate at a higher level of interest rates under a fiscally dominant regime. These results are summarized in Figures 9 and 10.

Figure 9 illustrates the relationship between optimal maturity and the short-term interest rate in a fiscally dominant regime, taking into account the presence of rollover risks. In the graph, the blue curve represents this relationship when there are no rollover risks, while the curve shifts to the right when rollover risks are present. The shift of the curve to the right signifies that the threshold, where the curve begins to rise, occurs at a lower level of r when rollover risks are present. This shift is due to the fact that when rollover risk exists, any increase in the interest rate results in higher interest costs for debt repayment in the current period. This increased cost makes it more challenging to meet principal repayments, which elevates rollover risk. Consequently, the Debt Management Office (DMO) must issue more short-term debt to cover these repayments, resulting in a positive relationship between α^*(optimal average maturity) and r. A similar relationship emerges

[21]Notice that in this case also, the first three terms are the same under no rollover risk or rollover risk ($B + C - Cov(r, G)$) and the only difference emanates because of the presence of the last term which changes sign depending on the presence of rollover risk.

[22]Notice that $[B + C - Cov(r, G)]^{FiscalDominance} > [B + C - Cov(r, G)]^{MonetaryDominance}$ because of the sign of the last term. Therefore, both the thresholds under the monetary dominance regime will be lower than that under the fiscal dominance regime.

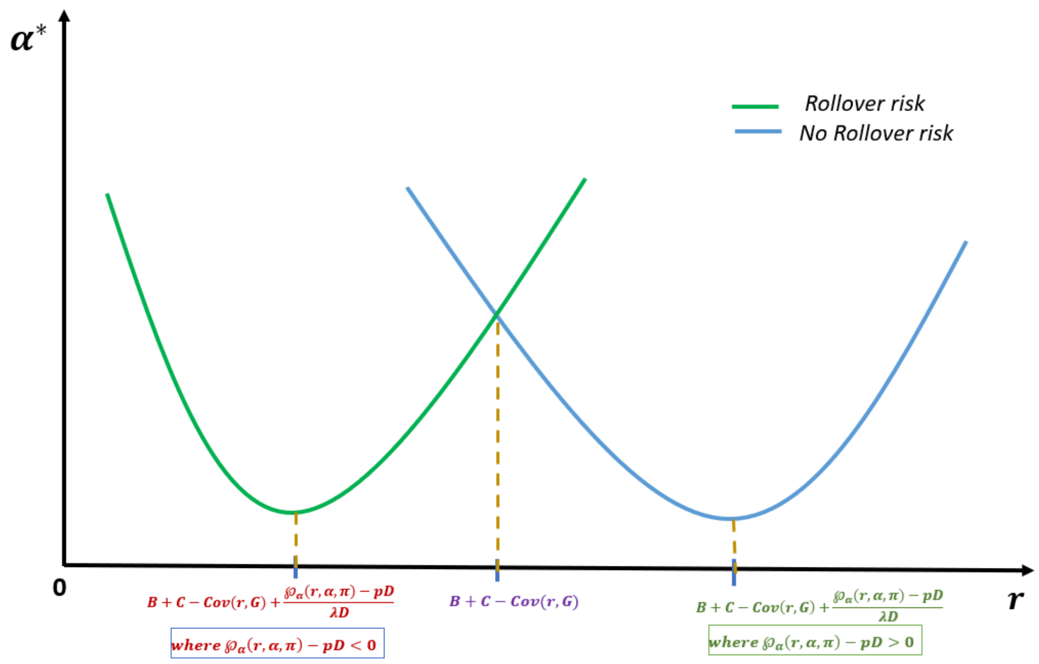

Figure 9: $\dfrac{\partial \alpha^*}{\partial r}$ under fiscal dominant regime

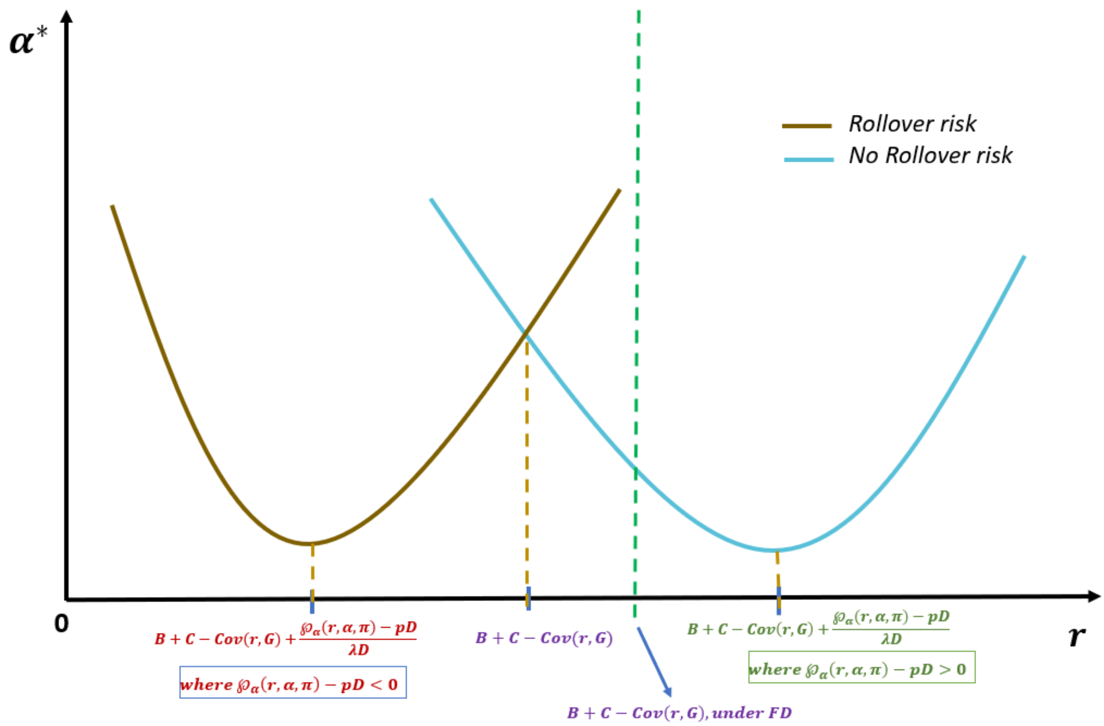

Figure 10: $\dfrac{\partial \alpha^*}{\partial r}$ under monetary dominant regime

under monetary dominance, as depicted in Figure 10. This similarity arises because of the symmetrical nature of the terms influencing the threshold.

However, it's important to note that despite the visual similarities in the relationships under both regimes, the threshold levels at which the curve begins to rise differ. Under fiscal dominance, the threshold is higher compared to monetary dominance. This is because, under fiscal dominance, the DMO reduces the average maturity of debt (lower α^*) to allow the fiscal authority to defer repayments to the future and meet its expenditures. This trend persists regardless of the presence of rollover risk.

When interest rates are sufficiently low, the DMO opts to "lock in" the lower rates by issuing longer-term bonds, anticipating future interest rate increases. This expectation intensifies in a fiscally dominant regime, as the central bank is more inclined to shift inflation into the future to maintain the real market value of debt. Conversely, under monetary dominance, the DMO reacts more swiftly. If there's any repayment risk, it increases the issuance of short-term debt, which is favored by investors. In summary, while the relationships appear visually similar under both regimes, the threshold levels at which the curves start to rise differ due to the distinct strategies employed by the DMO in response to varying levels of interest rates and repayment risks.

Our theoretical model provides us with an analytical framework for understanding how fiscal dominance impacts the maturity structure, with a focus on future inflation rather than current inflation. The two comparative statics exercises demonstrate the parametric conditions required under different regimes to generate the two channels through which fiscal dominance affects the maturity structure. We emphasize the significance of rollover risks in defining these thresholds and their role in shaping the entire dynamic in the second comparative statics analysis. To empirically investigate the presence of fiscal dominance through our proposed channel, we employ the decomposition methodology proposed by Hall and Sargent (2011) on our assembled granular-level dataset covering central government securities in India from 1999 to 2022. The next section provides the accounting decomposition and the results.

5 Data and Accounting Decomposition

To assess the presence or absence of fiscal and monetary coordination as outlined in our model, we employ the methodology proposed by Hall and Sargent (2011) on central government securities in India. Building on Das and Ghate (2022), we compile a unique dataset consisting of granular-level central government securities data spanning from 1999 to 2022. In our analysis, we introduce an additional term premium component. The subsequent subsection provides a brief overview of the data, while the following subsection introduces the decomposition methodology applied to this dataset

5.1 Data

We compile a comprehensive annual time series dataset on aggregate government debt spanning from 1999 to 2022, which encompasses its constituent elements. These components encompass the nominal interest rate, inflation, real GDP growth rate, and the primary deficit/surplus. At the security level, our data acquisition was based on the Ministry of Finance, Government of India's Status Papers, which meticulously record the total outstanding value of Central government securities for each fiscal year, alongside specifics about new issuances within that period. Our approach involves segmenting the securities into zero-coupon bonds through the separation of coupons. Subsequently, we formulated distinct matrices to capture principal payments and coupon payments. We focus on central government securities from 1999 to 2022.

Using data from the Reserve Bank of India (RBI), we obtain yield information from Subsidiary General Ledger (SGL) transactions relating to government-dated securities across various maturities. We use the March end yields for each year corresponding to each maturity in order to calculate the corresponding prices of bonds for a particular maturity. The month-end yields for government dated securities are given in Figure 11. an Using this yield curve data sourced from the RBI, we conducted price and market value calculations for the debt. Our analysis then undertake a debt decomposition following the methodology outlined in Hall and Sargent (2011). The other variables essential for the decomposition process, such as data for GDP, is sourced from various issues of the Economic Survey. Information about CPI inflation is accessed from the OECD, while data for the primary deficit/surplus is procured from the RBI. The detailed methodology for the creation of the dataset is given in Appendix E which also mentions the complete steps in assembling and preparing the dataset that we use in our decomposition.

5.2 Decomposition

We use the Hall and Sargent (2011) methodology to decompose the change in debt-GDP ratio for India using our security level dataset. This section provides the decomposition algorithm detailing the main steps undertaken. All the derivations are provided in Appendix A.

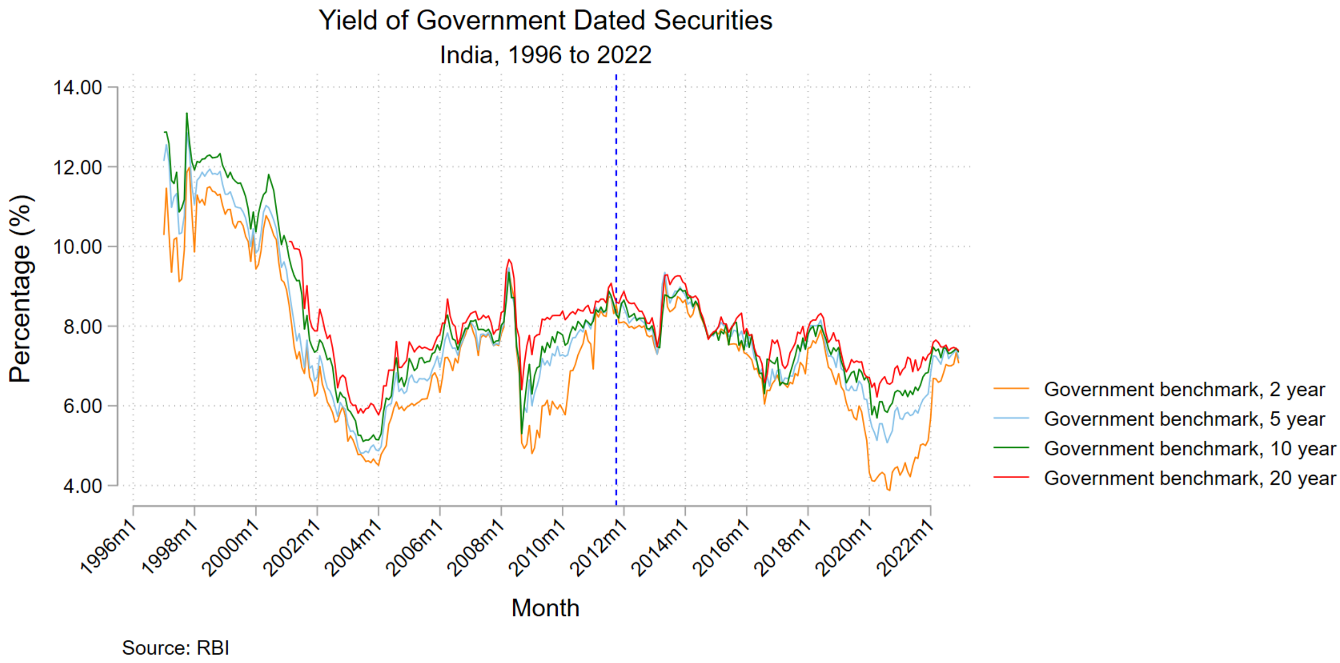

Figure 11: Month end Yields of Government Dated Securities

We start by writing the government budget constraint, which is given as

$$\frac{B_t}{Y_t} = (r_{t-1,t} - \pi_{t-1,t} - g_{t-1,t})\frac{B_{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t} + \frac{B_{t-1}}{Y_{t-1}} \tag{12}$$

where Y_t is real GDP at time t, B_t is the real value of securities issued by the fiscal authority, and $r_{t-1,t}, \pi_{t-1,t}, g_{t-1,t}$ are the nominal interest rate, inflation rate, and the growth rate of real GDP from period t-1 to period t respectively. We abstract away from inflation-indexed securities as well as securities denominated in foreign currency as the proportion of these securities is minimal ($\approx$ 4%). Equation 12 shows that the value of debt-to-GDP in time period t is equal to the sum of the nominal interest rate payments net of growth and inflation (the first term on the RHS) and the deficit-to-GDP ratio of this time period and the debt-to-GDP ratio of the previous time period (second and third terms respectively).

To explain the implications of interest rate risk and the debt's maturity structure on Debt-GDP ratio's evolution, we redefine the budget constraint. Let $\tilde{B}^j_{t-1}$ be real values of nominal zero-coupon bonds of maturity *j at t-1*. $\tilde{r}^j_{t-1,t}$ is the net nominal holding period return between *t-1 and t* on nominal zero-coupon bonds of maturity *j*. Using these definitions the new redefined budget constraint is given as

$$\frac{\tilde{B}_t}{Y_t} = \sum_{j=1}^{n} \tilde{r}^j_{t-1,t} \frac{\tilde{B}^j_{t-1}}{Y_{t-1}} - (\pi_{t-1,t} + g_{t-1,t}) \frac{\tilde{B}_{t-1}}{Y_{t-1}} \\ + \frac{def_t}{Y_t} + \frac{\tilde{B}_{t-1}}{Y_{t-1}} \tag{13}$$

The equation above differentiates between the contributions to the growth of the debt-GDP ratio that are contingent on the maturity of debt j, specifically the nominal interest payments, and those that are not. Next, at each date t we compute the number of rupees the government has promised to pay at each date $t + j$. The coupons are stripped from the coupon bonds and they are valued as a weighted sum of zeroes as any coupon bond can be decomposed into zero coupon bonds with varying maturity.

Let s^t_{t+j} be the number of time $t + j$ rupees that the government has at time t promised to deliver while q^t_{t+j} be the number of time t dollars that it takes to buy a dollar at time t+j, so this is like the price of a bond. q^t_{t+j} is given by

$$q^t_{t+j} = \frac{1}{(1 + \rho_{jt})^j}$$

where ρ_{jt} the time t yield to maturity (YTM) on bonds with j periods to maturity. We obtained the YTM data from RBI to compute prices. Since we are interested in the real market value, we define the deflator as $v_t = 1/p_t$, with p_t being the price level in base year t[23]. Therefore, with these modifications, we get the total real value of government debt outstanding in period t as

$$v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j}$$

Hence, we can write the redefined time t budget constraint as

$$v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j} = v_t \sum_{j=1}^{n} q^t_{t+j-1} s^{t-1}_{t+j-1} + def_t \tag{14}$$

Equation 14 denotes the real value of the government budget constraint taking into account the different tranches of the maturity structure of the debt that the government has issued. It shows the real value of the interest-bearing debt at the end of period t, given by the left hand of equation 14, is equal to the real value of the government's outstanding debt at the beginning of period t and the primary deficit-GDP ratio in period t. All the calculations are provided in Appendix B.

[23] This says that in order to convert 1 rupee to the equivalent amount of good is given by $v_t = 1/p_t$

In order to draw an analogy between equations 13 and 14, we divide both side of 14 by Y_t and rewrite as

$$\frac{v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}}{Y_t} = \sum_{j=1}^{n} \left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \frac{v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t} + \frac{v_{t-1} \sum_{j=1}^{n} q_{t+j-1}^{t} s_{t+j-1}^{t-1}}{Y_{t-1}} \tag{15}$$

Mapping these terms we can see that $v_t q_{t+j}^{t} s_{t+j}^{t} = \tilde{B}_t^j$ is the value of debt of maturity j at time t. For all maturities we will get the value of debt in period t as,

$$\tilde{B}_{t-1} = \sum_{j=1}^{n} \tilde{B}_{t-1}^{j}$$

Since $v_t = 1/p_t$,

$$\left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) = \tilde{r}_{t-1,t}^{j} - \pi_{t-1,t} - g_{t-1,t} \tag{16}$$

Defining terms in this way we see the clear mapping between equations 13 and 14.

5.3 Introducing a Term Premium

In this paper we extend the Hall and Sargent (2011) approach to separate the short rate and the term premium. Given that q_{t+j}^{t} is the number of time t rupees that it takes to buy a rupee at time t+j, this is like the price of a zero coupon bond with maturity j. Therefore, by definition, the net nominal holding period return between time *t-1 and t* on nominal zero-coupon bonds of maturity j is given as

$$\tilde{r}_{t-1,t}^{j} = \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} + 1$$

which can be interpreted as buying a j period bond at time $t-1$ and selling it as a $j-1$ year bond at time t. Therefore, following Cochrane and Piazzesi (2005) and Begenau et al. (2015) we can express this holding period return into two components: the short rate which is like the 1-year YTM, and the excess returns or the term premium

$$\tilde{r}_{t-1,t}^{j} = \rho_{1t} + \tilde{rx}_{t-1,t}^{j} \tag{17}$$

where

ρ_{1t}: Short rate or the date t yield to maturity on an 1-period bond

$\tilde{rx}^{j}_{t-1,t}$: excess return holding the j period bond from date *t-1* to date t

Thus we can write

$$\left(\frac{v_t}{v_{t-1}}\frac{q^t_{t+j-1}}{q^{t-1}_{t+j-1}}\frac{Y_{t-1}}{Y_t}-1\right)\approx(\rho_{1t}+\tilde{rx}^{j}_{t-1,t})-\pi_{t-1,t}-g_{t-1,t} \tag{18}$$

where we have augmented the initial Hall and Sargent (2011) equation with the term premium explicitly appearing in it.

5.4 Contributions of inflation, growth and term premium for debt decomposition

To take into account the role played by inflation, growth, and nominal returns including the term premium component, we write equation 13 as

$$\begin{aligned}\frac{\tilde{B}_t}{Y_t}=&\sum_{j=1}^{n}(\rho_{1t}+\tilde{rx}^{j}_{t-1,t})\frac{\tilde{B}^j_{t-1}}{Y_{t-1}}-(\pi_{t-1,t}+g_{t-1,t})\frac{\tilde{B}_{t-1}}{Y_{t-1}}\\&+\frac{def_t}{Y_t}+\frac{\tilde{B}_{t-1}}{Y_{t-1}}\end{aligned} \tag{19}$$

To explicitly consider the role played by different factors in shaping the evolution of the debt-GDP ratio during various time periods, we iterate the above equation backward to an initial time $t-\tau$,

$$\begin{aligned}\frac{\tilde{B}_t}{Y_t}-\frac{\tilde{B}_{t-\tau}}{Y_{t-\tau}}=&\sum_{s=0}^{\tau-1}\left[\sum_{j=1}^{n}(\rho_{1t-s-1}+\tilde{rx}^{j}_{t-s-1,t-s}-\pi_{t-s-1,t-s}-g_{t-s-1,t-s})\frac{\tilde{B}^j_{t-s-1}}{Y_{t-s-1}}\right.\\&\left.+\frac{def_{t-s}}{Y_{t-s}}\right]\end{aligned} \tag{20}$$

The equation above illustrates that commencing from an initial time frame, we can characterize the evolution of the debt-to-GDP ratio through the decomposition of the terms present on the right-hand side of equation 20. Note that the terms on the right side of the equation are actually weighted by the debt-to-GDP ratio of various maturity tranches. This suggests that the factors contributing to the debt-to-GDP evolution—specifically the nominal return (a combination of term premium and short rate) net of inflation and the growth rate—are influenced by the issuance of debt-to-GDP for each maturity within a given yearly tranche.

Appendix B provides all the calculations for arriving at this equation as well as an example of the decomposition equation applied to a particular time frame, demonstrating how changes in the debt-to-GDP ratio can be deconstructed. Furthermore, Appendix B outlines the utilization of the term premium in an example involving two types of bonds. Equation 20 is the key equation that we take to our assembled dataset to analyze the evolution of debt and what role monetary-fiscal interaction play in it.

6 Debt Decomposition Results

Table 2 presents our decomposition findings based on the compiled data, utilizing equation 20. Our analysis is divided into distinct sub-periods, each reflecting the evolution of the debt-GDP ratio over time. Across the entire sample (1999-2022), the debt-GDP ratio grew by 30.8 percentage points. This increase was driven predominantly by the nominal return component, particularly at the short-rate segment, which exerted a significant upward influence on debt. Conversely, inflation and economic growth contributed to mitigating the rise, thereby reducing the debt-GDP ratio. Notably, bondholders realized a net positive real return, as is evident from the positive real return component in the first column.

Upon dissecting the decomposition for various sub-periods, a recurring pattern emerges. Across most sub-periods, the dominant factors influencing the change in debt-GDP ratio remain the nominal return and the inflation components. With the exception of the 2004-2014 span, the debt-GDP ratio increased in all other sub-periods. Notably, in the years following the COVID pandemic, the debt-GDP increased by approximately 13 percentage points in just couple of years. While again nominal return and inflation are the major components, negative growth rate of GDP actually led to an increase in the real value of debt-GDP.[24] The key takeaway from our decomposition result is the important role played by the nominal interest rates and inflation in the evolution of debt-GDP ratio in India.

The outcomes of the security-level decomposition, structured by maturity tranches, are illustrated in Table 3. We categorize the analysis into three maturity ranges: 1-2 years, 2-10 years, and 10+ years. A salient observation from Table 3 is that inflation and the short-rate component collectively contribute to more than half of the changes in the debt-GDP ratio across the entire sample period and within the delineated sub-periods, particularly within the longer maturity tranches. This shows that short-rate plays an important role in impacting the maturity structure of debt (our second channel). Notably, in the aftermath of the global financial crisis (2009-2014), holders of long-term bonds encountered negative real returns—a trend that also emerged in the post-COVID era (2020-2022). Following the adoption of the inflation-targeting regime (2014-2020), the contribution of the inflation component in reducing debt-GDP reduced, but it regained prominence in the immediate aftermath of the COVID pandemic (2020-2022).

[24] The component of growth rate is positive which denotes that negative growth during these years actually led to an increase in the debt-GDP rather than reducing it.

The results highlight the substantial role played by the short-rate component in the above mentioned maturity tranches, contributing approximately 12.2% of the total shift in the debt-GDP ratio. The term premium component, accounting for about 12% of the total change in debt-GDP during the entire sample period, underscores the influence of market expectations, risk considerations, and liquidity factors that impact the valuation of long-term bonds. It is pertinent to note that due to data limitations, the explicit separation of these contributing factors isn't feasible; thus, these results show the collective contribution of all of these components.

The key takeaway from these decomposition results underscores the substantial influence exerted by inflation and nominal interest rates. While inflation significantly contributes to reducing the debt-GDP ratio, its impact becomes more pronounced when the maturity structure of debt is extensive, particularly for longer-term securities. Nominal interest rates, represented by the short-rate component, which we use as a proxy for the central bank's policy rate, also assumes a pivotal role in driving up the debt-GDP ratio. Consequently, the role of inflation becomes vital in maintaining the real value of debt at a sustainable level.

However, when the central bank is committed to containing inflation within a specified range, a mere escalation of price levels may not suffice to ensure debt sustainability. Instead, the central bank can use the maturity structure of debt to defer the surge in prices to the future (leading to future unexpected inflation) and instead use the market value of debt to keep debt at a sustainable level in the current time period.

According to our model, a higher debt-GDP ratio has a positive impact on the maturity structure of debt. Our debt decomposition results illustrate this phenomenon, as the effect of changes in the debt-GDP ratio is more pronounced in the upper maturity tranches than in the short-term tranche (1-2 years). Across the entire sample period, out of the 30.8% increase in the debt-GDP ratio, the majority of the contribution in each sub-component comes from the higher maturity tranche. In the following subsection, we present a time series that demonstrates this positive correlation, compiled from our data.

Our second channel of fiscal dominance, which involves the short-term interest rate affecting the maturity structure, is also evident in our results. When we isolate the term premium component, it becomes clear that, regardless of the sub-period analyzed, the short-term interest rate impacts the longer maturity structure. Both propositions of our model consider an important component: the presence or absence of rollover risk. We are currently working on decomposing the term for rollover risk in our exercise to further explore how fiscal dominance operates in that scenario.

Therefore, the debt decomposition results demonstrates the presence of fiscal dominance in India operating through the channel of the maturity structure of debt. This phenomenon gained prominence after 2015 when India adopted the inflation targeting regime. In the context of an inflation-targeting central bank, the mandate to maintain inflation within a specified band necessitates alternative means to keep the government debt at a sustainable level, especially when government spending is on the rise. This alternative channel is the maturity structure of debt, which effectively shifts the burden of inflation from the present to the future when the debt matures and requires repayment. In this way, it keeps inflation in check in the present while leveraging it as a tool for the future[25]. Next we present some additional results from our assembled dataset which shows the time series of the key comparative statics, namely the relationship between debt-GDP ratio and short-rate with the average maturity of debt.

6.1 Additional results

Our analysis also reveals notable relationships between the average maturity of debt and key economic indicators. Specifically, we observe a positive correlation between the average maturity of debt and the debt-GDP ratio (figure 12), as well as a negative correlation with the short-rate (figure 13).

While the former relationship can be explained by the strategy of minimizing risks tied to rolling over mounting debt, our static model demonstrates an interesting phenomenon. We find that even in scenarios devoid of rollover risk, the Debt Management Office (DMO) tends to increase the average maturity of debt alongside rising debt levels. This behavior becomes particularly pronounced when the DMO aims to secure the current lower interest rates as a preemptive measure against anticipated future rate hikes. Given that approximately 95% of India's issued debt comprises fixed-rate bonds, this approach offers a valid strategy to capitalize on existing lower interest rates. Our formal model further substantiates that this strategy is more prevalent in a fiscally dominant regime.

Our second finding about negative relation between the short-rate and average maturity of debt (figure 13) is interesting. Intuitively, a reduction in the short-rate should logically prompt an increase in the issuance of short-term debt, enabling the government to

[25]Cochrane (2022a) looks at the role of maturity structure in the US context using the *face value* of debt. While he also found the growing role of future inflation but our approach is different in that we use the market value of debt which takes into account the capital gains or losses explicitly into the picture. Thus, any change in the interest rate, which might happen as a result of inflation, will have an impact on the market value. Moreover, the debt valuation equation given by 7 takes the market value of debt on the right-hand side. Thus our results using the market value of debt will take those effects into account as well.

capitalize on lower interest payments. However, our analysis reveals an opposing relationship. Our decomposition analysis reinforces this finding. It indicates that the impact of the short-rate component on debt dynamics is more pronounced for longer maturity tranches than for shorter ones. This highlights that the short-rate holds particular significance for investors engaged in longer-term securities.

Our formal model demonstrates that this phenomenon is more likely to occur in a fiscally dominant regime, especially when rollover risk is absent. In summary, our findings underscore the intricate interplay between fiscal dominance, maturity structure, and interest rates in shaping debt dynamics, with implications extending to risk management.

Table 2: Debt decomposition for Central Government Securities

		Period					
Start **End**		**2000-** **2022**	**2000-** **2004**	**2004-** **2009**	**2009-** **2014**	**2014-** **2020**	**2020-** **2022**
Debt-GDP							
	Start	19.70	19.70	35.60	34.90	34.70	37.30
	End	50.50	35.60	34.90	34.70	37.30	50.50
	Change	30.80	15.90	-0.70	-0.20	2.60	13.20
Marketable debt							
	Nominal return	55.80	15.10	8.40	9.40	13.60	9.40
	Short-rate	45.60	9.10	8.50	12.10	9.90	5.90
	Ex_Return(TP)	14.10	6.90	-0.10	-2.70	4.20	3.50
	Inflation	-41.30	-5.60	-7.70	-15.70	-6.40	-6.00
	Real return	14.50	9.10	0.60	-6.30	6.80	3.40
	Growth rate	-32.00	-8.90	-9.00	-7.00	-8.10	1.00
Non-marketable debt							
	Nominal return	29.70	13.30	7.40	4.10	3.10	1.70
	Inflation	-5.40	-0.40	-1.10	-2.10	-0.70	-1.10
	Growth rate	-3.60	-0.80	-1.30	-0.90	-0.90	0.30
Primary Deficit/GDP		26.00	3.40	1.90	10.80	2.30	7.70

This table presents the outcomes of the decomposition analysis conducted using equation 20 on our collected data. It outlines the evolution of the debt-GDP ratio for each specified time period, accompanied by an analysis of the contributing factors that have influenced the changes observed in the debt-GDP ratio.
Source: Author's calculations

Table 3: Debt decomposition for Central Government securities by maturity structure

			Period					
Start **End**			**2000-** **2022**	**2000-** **2004**	**2004-** **2009**	**2009-** **2014**	**2014-** **2020**	**2020-** **2022**
Debt-GDP								
	Start		19.70	19.70	35.60	34.90	34.70	37.30
	End		50.50	35.60	34.90	34.70	37.30	50.50
	Change		30.80	15.90	-0.70	-0.20	2.60	13.20
Marketable debt								
	Nominal return		55.80	15.10	8.40	9.40	13.60	9.40
		1-2 years	13.80	3.00	2.60	3.20	3.10	1.90
		TP(1-2 Years)	1.60	0.60	0.30	-0.2	0.40	0.50
		2 -10 years	30.01	8.80	4.40	5.00	6.90	4.90
		TP(2-10 yrs)	8.20	4.40	0.40	-1.1	2.00	2.50
		10+ years	12.20	3.30	1.30	1.20	3.60	2.60
		TP(10+ yrs)	4.30	1.90	-0.20	-0.7	1.70	1.60
	Inflation		-41.3	-5.60	-7.70	-15.70	-6.40	-6.00
		1-2 years	-12.30	-1.60	-2.40	-4.70	-1.90	-1.70
		2 -10 years	-21.60	-3.10	-3.90	-8.30	-3.30	-3.00
		10+ years	-7.50	-1.00	-1.40	-2.60	-1.20	-1.20
	Growth rate		-32.00	-8.90	-9.00	-7.00	-8.10	1.00
		1-2 years	-9.00	-2.40	-2.60	-2.00	-2.3 0	0.30
		2 -10 years	-16.80	-4.80	-4.60	-3.80	-4.20	0.5
		10+ years	-6.20	-1.80	-1.80	-1.20	-1.60	0.2
Non-marketable debt								
	Nominal return		29.70	13.30	7.40	4.10	3.10	1.70
	Inflation		-5.40	-0.40	-1.10	-2.10	-0.70	-1.10
	Growth rate		-3.60	-0.80	-1.30	-0.90	-0.90	0.30
	Primary Deficit to GDP		26.00	3.40	1.90	10.80	2.30	7.70

This table presents the outcomes of the decomposition analysis by further bifurcating it maturity wise. Given the important role played by the maturity structure of debt in our study, we extended our analysis across the same time periods as indicated in table 2, while also explicitly incorporating the maturity structure. This approach enabled us to discern the contributions of different components within various maturity tranches, thereby influencing the overall debt-GDP ratio.
Source: Author's calculations

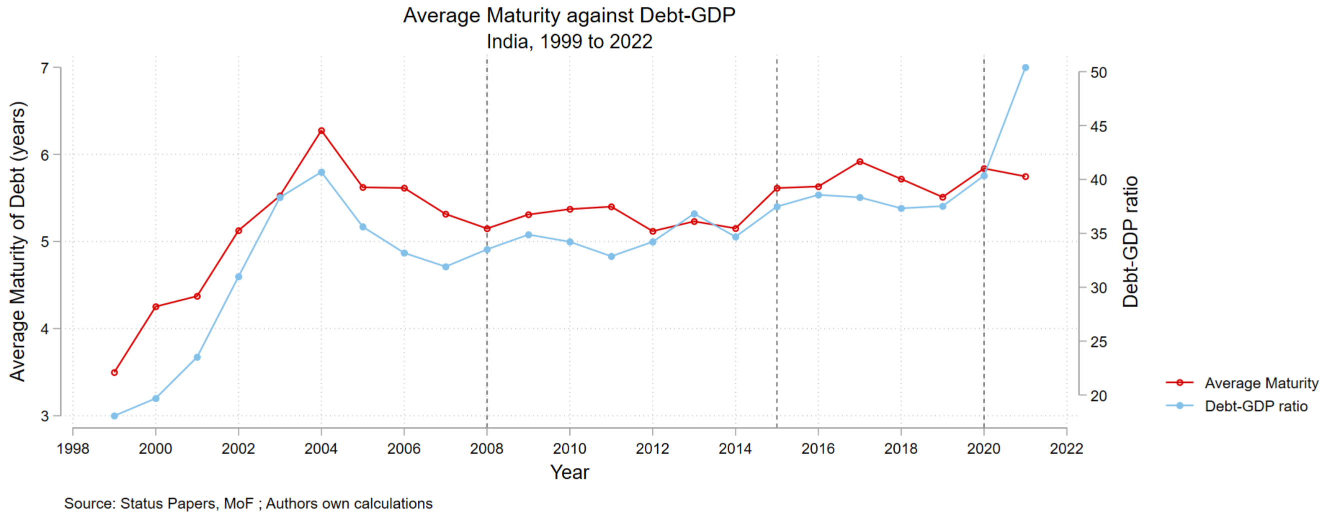

Figure 12: Average Maturity against the Debt-GDP ratio

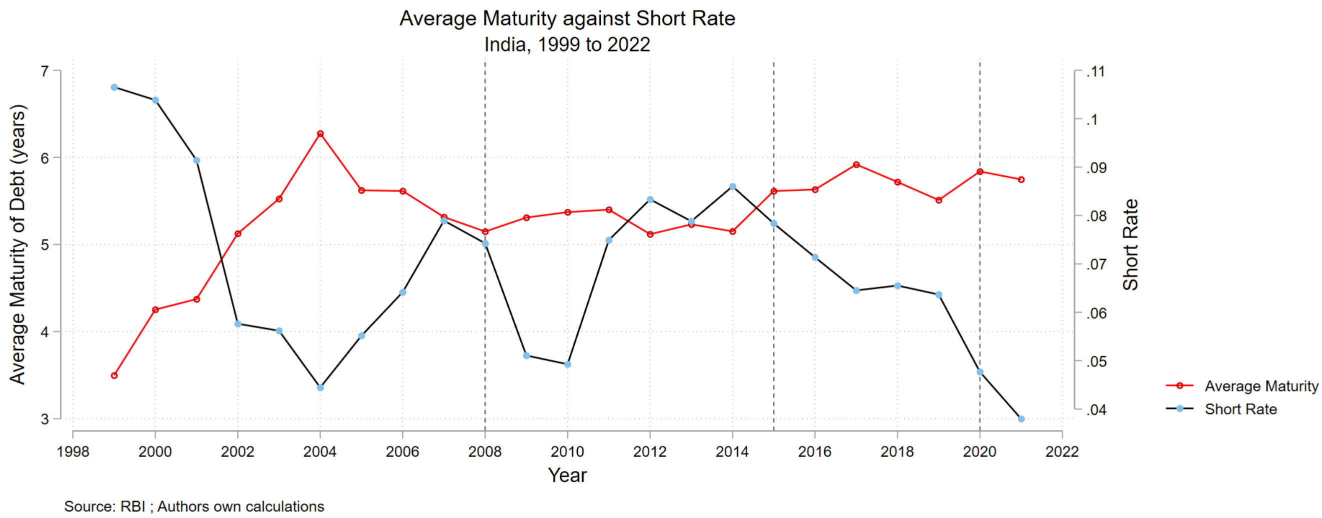

Figure 13: Average Maturity against the short-rate

7 Conclusion

The surge in sovereign debt on a global scale, post the Great Financial Crisis and especially after the COVID-19 pandemic, necessitates a comprehensive exploration of the factors that can alleviate the burden of debt. Factors like the nominal interest rates, economic growth, inflation, and the primary deficit/surplus play a pivotal role in influencing changes in the debt-GDP ratio. However, the intricate interplay of these factors necessitates a deeper understanding of their dynamics.

In addition to the factors mentioned above, the dynamics of sovereign debt are intricately linked with the coordination between monetary and fiscal authorities. When one of these authorities takes on a "dominant" role, the instruments and mechanisms influencing debt dynamics can significantly shift. This paper introduces a novel channel of monetary-fiscal coordination, which complements existing frameworks, in evaluating debt dynamics. Specifically we examine the maturity structure of debt, which provides a nuanced lens through which to understand and assess debt sustainability.

The uniqueness of India's economic context adds further weight to the importance of considering the maturity structure of debt. With ceilings imposed on inflation and deficits due to Flexible Inflation Targeting and the Fiscal Responsibility and Budget Management Act, the roles of nominal interest rates and GDP growth rate have become important in shaping the debt dynamics.

Additionally, India's central bank's dual responsibilities of managing public debt and conducting monetary policy magnifies the influence of the maturity structure on both monetary policy and debt management. Our research not only sheds light on the impact of fiscal dominance on debt-GDP dynamics but also presents a comprehensive empirical analysis using a novel dataset. By dissecting the outcomes according to distinct maturity segments, we uncover important relationships between average maturity of debt, short-term rates, and inflation. These findings underline the intricate interplay of various factors in shaping debt dynamics and have significant implications for both debt management and monetary policy.

Furthermore, our static model provides a theoretical foundation for understanding how the Debt Management Office (DMO) optimally structures the maturity of debt under different regimes and risk scenarios. This insight adds depth to our empirical findings and contributes to a comprehensive understanding of the interconnections between fiscal policy, monetary policy, and debt management.

Our paper contributes to the ongoing discourse on debt sustainability by examining the role of maturity structure in influencing debt-GDP dynamics. The empirical insights, combined with the theoretical model, provide a holistic understanding of the factors at play in managing sovereign debt in the Indian context. As governments across the world grapple with the challenges of rising debt levels, this study's findings hold relevance for policymakers and researchers aiming to foster economic stability and fiscal sustainability.

Work in Progress

As the next step, it is imperative to introduce a term for the rollover risk component. Our theoretical model has shown that the thresholds we identified depend significantly on the presence of rollover risk. Therefore, incorporating an explicit term for rollover risk into our decomposition is crucial. Furthermore, it's important to note that our theoretical model is a partial equilibrium static model, which currently does not account for the general equilibrium effects of the Debt Management Office's (DMO) choices. Specifically, the issue of debt ownership is a critical consideration that we have temporarily abstracted from in our current analysis[26].

[26] We assumed in our model that the debt that is issued is fully demanded by the agents (like Greenwood et al. (2015)) without accounting for who is holding the debt, which is going to be important from the perspective of fiscal dominance

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Online Appendix

Appendix A: Figures

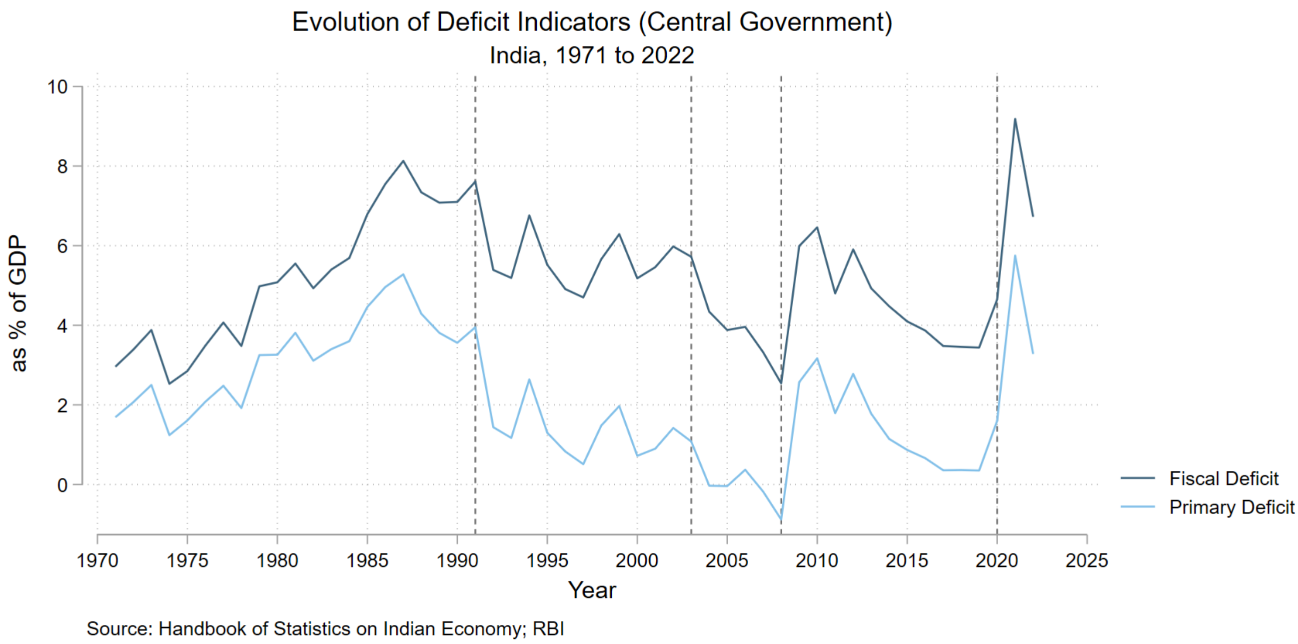

Figure 14: Evolution of fiscal and primary deficit (as % of GDP) for central government

Note: This figure shows the evolution of primary and fiscal deficit for the central government from 1971 to 2022. The first event in 1991 marks the economic liberalization policies which saw measures to reduce the debt of the government accumulated in the 1980s. The second major event was in 2003 with the passage of the FRBM Act that aims to control and manage the fiscal deficit of the government, ensuring fiscal discipline and sustainability by setting targets and limits on government borrowing and expenditure. The government spending escalated after both the crises (in 2008 and 2020).

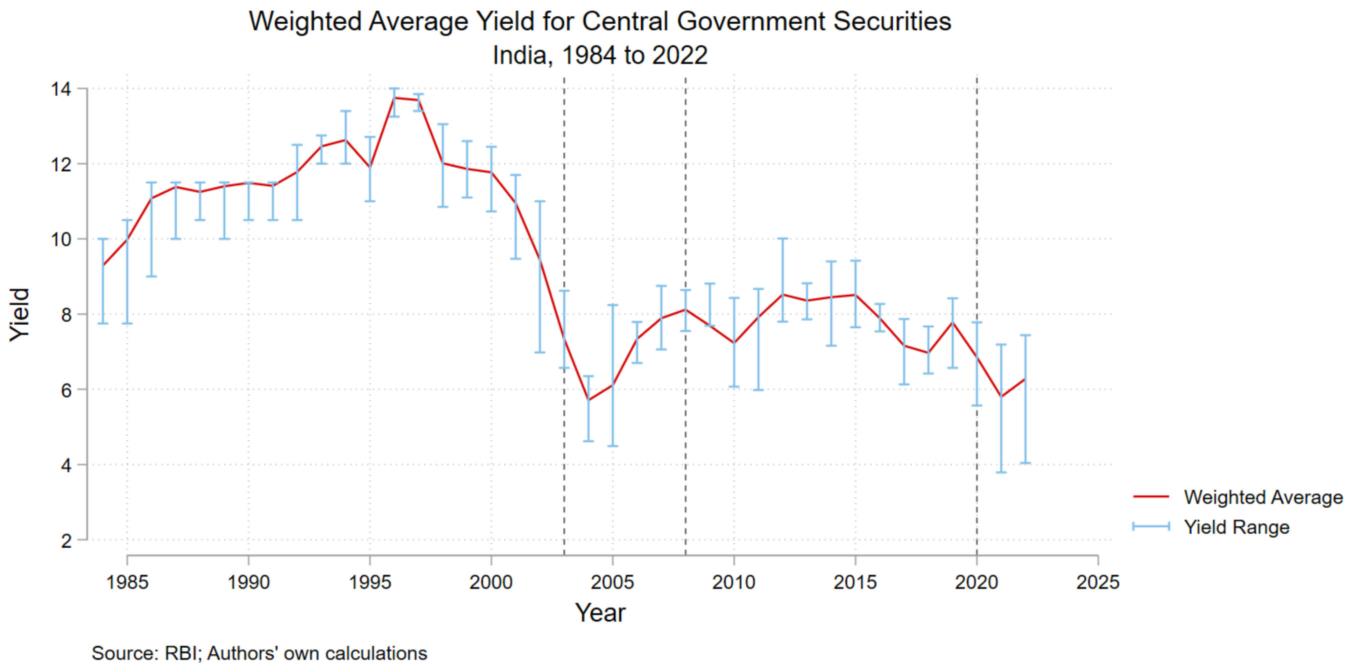

Figure 15: Weighted Average Yields for Central Government Securities

Note: This figure plots the weighted average yields of central government securities for each year. The bars around each point shows the lower and upper bounds of yield for any particular year. Post 2000 there has been a significant drop in the yields for these securities. The length of each bar represents the range of yields provided.

Appendix B: Derivations of the Decomposition Equation

In this appendix, we derive equation (4) of the paper detailing all the steps in the process.

We start by writing the government budget constraint, which is given as

$$\frac{B_t}{Y_t} = (r_{t-1,t} - \pi_{t-1,t} - g_{t-1,t})\frac{B_{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t} + \frac{B_{t-1}}{Y_{t-1}} \tag{21}$$

To bring some of the consequences of interest rate risk and the maturity structure of the debt for the evolution of the Debt-GDP ratio, the budget constraint is redefined.

Let $\tilde{B}^j_{t-1}$ be real values of nominal zero-coupon bonds of maturity *j at t-1*.

$\tilde{r}^j_{t-1,t}$ is the net nominal holding period return betweent *t-1 and t* on nominal zero-coupon bonds of maturity *j*.

Then using these definitions the new redefined budget constraint is given as

$$\begin{aligned}\frac{\tilde{B}_t}{Y_t} = & \sum_{j=1}^{n} \tilde{r}^j_{t-1,t}\frac{\tilde{B}^j_{t-1}}{Y_{t-1}} - (\pi_{t-1,t} + g_{t-1,t})\frac{\tilde{B}_{t-1}}{Y_{t-1}} \\ & + \frac{def_t}{Y_t} + \frac{\tilde{B}_{t-1}}{Y_{t-1}}\end{aligned} \tag{22}$$

The above equation distinguishes contributions to the growth of the debt-GDP ratio that depend on debt maturity j from those that don't.

Accounting Details

To Carry out the accounting details, At each date, t, compute the number of rupees the government has promised to pay at each date t+j. The coupons are stripped from the coupon bonds and they are vallued as weighted sum of zeroes as any coupon bond can be decomposed into zero coupon bonds with varying maturity.

s^t_{t+j} is the number of time t+j dollars that the government has at time t promised to deliver.

q^t_{t+j} be the number of time t dollars that it takes to buy a dollar at time t+j, so this is like the price of a bond.

$$q^t_{t+j} = \frac{1}{(1+\rho_{jt})^j}$$

where ρ_{jt} the time *t* yield to maturity on bonds with *j* periods to maturity.
To convert t dollars to goods, use $v_t = 1/p_t$, with p_t being the price level in base year.

The total real value of government debt outstanding in period t equals

$$v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}$$

Thus now we can define the time t budget constraint as

$$v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t} = v_t \sum_{j=1}^{n} q_{t+j-1}^{t} s_{t+j-1}^{t-1} + def_t \tag{23}$$

The left-hand side of equation (2) is the real value of the interest bearing debt at the end of period t. The right-hand side of equation (2) is the sum of the real value of the primary deficit and the real value of the outstanding debt that the government owes at the beginning of the period.

Now to attain the government budget constraint that we have in (**??**),we will proceed as follows. Using equation (2),

$$v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t} = v_t \sum_{j=1}^{n} q_{t+j-1}^{t} s_{t+j-1}^{t-1} + def_t$$

Divide by Y_t on both sides to get

$$\frac{v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}}{Y_t} = \frac{v_t \sum_{j=1}^{n} q_{t+j-1}^{t} s_{t+j-1}^{t-1}}{Y_t} + \frac{def_t}{Y_t}$$

Now add and subtract $\frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}}$ on the RHS of the above equation, we get

$$\begin{aligned}\frac{v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}}{Y_t} =& \frac{\sum_{j=1}^{n} v_t q_{t+j-1}^{t} s_{t+j-1}^{t-1}}{Y_t} - \frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}} \\ &+ \frac{def_t}{Y_t} + \frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}}\end{aligned}$$

$$\begin{aligned}\frac{v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}}{Y_t} + =& \left(\frac{\frac{\sum_{j=1}^{n} v_t q_{t+j-1}^{t} s_{t+j-1}^{t-1}}{Y_t}}{\frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}}} - 1 \right) \frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}} \\ &+ \frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t}\end{aligned}$$

Cancelling the terms and rearranging, we get

$$\frac{v_t \sum_{j=1}^{n} q_{t+j}^{t} s_{t+j}^{t}}{Y_t} = \sum_{j=1}^{n} \left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \frac{v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t} + \frac{v_{t-1} \sum_{j=1}^{n} q_{t+j-1}^{t} s_{t+j-1}^{t-1}}{Y_{t-1}} \tag{24}$$

To recognize the terms in the above equation and see the analogy with equation number (2),we see that

$$v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1} = \tilde{B}_{t-1}^{j}$$

$$\tilde{B}_{t-1} = \sum_{j=1}^{n} \tilde{B}_{t-1}^{j}$$

Now since $v_t = 1/p_t$, thus we will have

$$\left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) = \tilde{r}_{t-1,t}^{j} - \pi_{t-1,t} - g_{t-1,t} \tag{25}$$

Deciphering the term premium

Now according to the given definitions, q_{t+j}^{t} is the number of time t dollars that it takes to buy a dollar at time t+j, so this is like the price of a zero coupon bond with maturity j.

Thus we can define the return on a *j* period bond one period later is given as $\tilde{r}_{t-1,t}^{j}$, which is the net nominal holding period return between time *t-1 and t* on nominal zero-coupon bonds of maturity *j*.

$$\tilde{r}_{t-1,t}^{j} = \frac{q_{t+j-1}^{t}}{q_{t+j-1}^{t-1}} + 1$$

The above equation which is the holding period return between *t-1* and *t* on nominal zero-coupon bonds of maturity *j* can be interpreted as buying a *j* period bond at time *t-1* and selling it as a *j-1* year bond at time *t*.

We can decompose the log return on an *j-period* bond one period later as

$$\tilde{r}_{t-1,t}^{j} = \rho_{1t} + \tilde{rx}_{t-1,t}^{j} \tag{26}$$

where

ρ_{1t}: Short rate or the date *t* yield to maturity on an 1-period bond
$\tilde{rx}_{t-1,t}^{j}$: excess return holding the *j* period bond from date *t* to date *t+1*
This measure of *excess return* in our *term premium*.

With this decomposition, we will have

$$\left(\frac{v_t}{v_{t-1}}\frac{q^t_{t+j-1}}{q^{t-1}_{t+j-1}}\frac{Y_{t-1}}{Y_t}-1\right)\approx(\rho_{1t}+\tilde{rx}^j_{t-1,t})-\pi_{t-1,t}-g_{t-1,t} \tag{27}$$

For the excess return which is simply

$$\tilde{rx}^j_{t-1,t}=\tilde{r}^j_{t-1,t}-\rho_{1t}$$

we can calculate it for all the maturity of bonds that are being issued by the government. So can define a vector across maturity (without the j superscript) as consisting of

$$\tilde{\mathbf{rx}}_{\mathbf{t-1,t}}=[\tilde{rx}^2_{t-1,t},\tilde{rx}^3_{t-1,t},\tilde{rx}^4_{t-1,t}...]^T$$

By this decomposition of the one period holding priod return into the short rate and the excess return premia, we can capture the dynamics that the term premia plays.

Contributions of inflation, growth and term premia for debt decomposition

Equation (2) in this model, which depicts the evolution of the debt to GDP ratio can now be written to get the term premia explicitly as

$$\begin{aligned}\frac{\tilde{B}_t}{Y_t}=&\sum_{j=1}^{n}(\rho_{1t}+\tilde{rx}^j_{t-1,t})\frac{\tilde{B}^j_{t-1}}{Y_{t-1}}-(\pi_{t-1,t}+g_{t-1,t})\frac{\tilde{B}_{t-1}}{Y_{t-1}}\\&+\frac{def_t}{Y_t}+\frac{\tilde{B}_{t-1}}{Y_{t-1}}\end{aligned} \tag{28}$$

As a next step take $\frac{\tilde{B}_{t-\tau}}{Y_{t-\tau}}$ as an initial condition at time $t-\tau$.
Iterate on equation (10), we can write

$$\begin{aligned}\frac{\tilde{B}_{t-1}}{Y_t}=&\sum_{j=1}^{n}(\rho_{1t-1}+\tilde{rx}^j_{t-2,t-1})\frac{\tilde{B}^j_{t-2}}{Y_{t-2}}-(\pi_{t-2,t-1}+g_{t-2,t-1})\frac{\tilde{B}_{t-2}}{Y_{t-2}}\\&+\frac{def_{t-1}}{Y_{t-1}}+\frac{\tilde{B}_{t-2}}{Y_{t-2}}\end{aligned} \tag{29}$$

Plugging (10) into (9) and arranging, we have

$$
\begin{aligned}
\frac{\tilde{B}_t}{Y_t} = &\sum_{j=1}^{n}\left[(\rho_{1t} + \tilde{rx}^j_{t-1,t})\frac{\tilde{B}^j_{t-1}}{Y_{t-1}} + (\rho_{1t-1} + \tilde{rx}^j_{t-2,t-1})\frac{\tilde{B}^j_{t-2}}{Y_{t-2}}\right] \\
&- (\pi_{t-1,t} + g_{t-1,t})\frac{\tilde{B}_{t-1}}{Y_{t-1}} - (\pi_{t-2,t-1} + g_{t-2,t-1})\frac{\tilde{B}_{t-2}}{Y_{t-2}} \\
&+ \frac{def_t}{Y_t} + \frac{def_{t-1}}{Y_{t-1}} + \frac{\tilde{B}_{t-1}}{Y_{t-1}} + \frac{\tilde{B}_{t-2}}{Y_{t-2}}
\end{aligned} \tag{30}
$$

Thus by iterating and collecting terms, we will reach the following decomposition equation as

$$
\begin{aligned}
\frac{\tilde{B}_t}{Y_t} - \frac{\tilde{B}_{t-\tau}}{Y_{t-\tau}} = &\sum_{s=0}^{\tau-1}\Bigg[\sum_{j=1}^{n}(\rho_{1t-s-1} + \tilde{rx}^j_{t-s-1,t-s} - \pi_{t-s-1,t-s} - g_{t-s-1,t-s})\frac{\tilde{B}^j_{t-s-1}}{Y_{t-s-1}} \\
&+ \frac{def_{t-s}}{Y_{t-s}}\Bigg]
\end{aligned} \tag{31}
$$

Equation (22) is the key equation that will be used for the decomposition.

An example

To see things in a much more simpler way and to see an example of how equation 22 will be implemented, let's take a concrete example.

Suppose we want to see how the debt to GDP ratio evolved during the time period 2005-09. For this purpose the appropriate values of t and τ will be 2009 and 04 respectively as we are measuring the change in the Debt-GDP for 4 years. We will have $n = 30$ as the maximum maturity period bond available to us was for 30 years.

Thus with this formulation, equation 23 will become

$$
\begin{aligned}
\frac{\tilde{B}_{2009}}{Y_{2009}} - \frac{\tilde{B}_{2005}}{Y_{2005}} = &\sum_{s=0}^{3}\Bigg[\sum_{j=1}^{30}(\rho_{1,2009-s} + \tilde{rx}^j_{2009-s-1,t-s} - \pi_{2009-s-1,2009-s} - g_{2009-s-1,2009-s})\frac{\tilde{B}^j_{2009-s-1}}{Y_{2009-s-1}} \\
&+ \frac{def_{2009-s}}{Y_{2009-s}}\Bigg]
\end{aligned} \tag{32}
$$

Now opening the outer summation of the RHS of the above equation to see the impact of each year, we will have

$$
\begin{aligned}
\frac{\tilde{B}_{2009}}{Y_{2009}} - \frac{\tilde{B}_{2005}}{Y_{2005}} = & \left[\sum_{j=1}^{30} (\rho_{1,2008} + \tilde{rx}^{j}_{2008,2009} - \pi_{2008,2009} - g_{2008,2009}) \frac{\tilde{B}^{j}_{2008}}{Y_{2008}} \right. \\
& \left. + \frac{def_{2009}}{Y_{2009}} \right] + \left[\sum_{j=1}^{30} (\rho_{1,2007} + \tilde{rx}^{j}_{2007,2008} - \pi_{2007,2008} - g_{2007,2008}) \frac{\tilde{B}^{j}_{2007}}{Y_{2007}} \right. \\
& \left. + \frac{def_{2008}}{Y_{2008}} \right] + \left[\sum_{j=1}^{30} (\rho_{1,2006} + \tilde{rx}^{j}_{2006,2007} - \pi_{2006,2007} - g_{2006,2007}) \frac{\tilde{B}^{j}_{2006}}{Y_{2006}} \right. \\
& \left. + \frac{def_{2007}}{Y_{2007}} \right] + \left[\sum_{j=1}^{30} (\rho_{1,2005} + \tilde{rx}^{j}_{2005,2006} - \pi_{2005,2006} - g_{2005,2006}) \frac{\tilde{B}^{j}_{2005}}{Y_{2005}} \right. \\
& \left. + \frac{def_{2006}}{Y_{2006}} \right] \qquad (33)
\end{aligned}
$$

So equation 24 gives decomposes the total change in the debt to GDP between the period 2005 to 2009 into the individual changes pertaining to each year.

7.1 Note on how average maturity of debt is calculated

Since average maturity plays an important role in our discussion, it's quite pertinent to give a brief discussion as to how it is calculated in our framework. The way Hall and Sargent (2011) calculates the average maturity or duration of the bonds is different from how the government calculates duration. The framework that Hall and Sargent (2011) proposes and we adopt is that we look at what is the per period average maturity of the debt that is outstanding in a given year taking into account that the maturity of a bond already issued changes every period and thus the payment outstanding also changes. We first show how the government calculates the average maturity and then show how we are doing this and what are the key differences between these two methodologies and how the method we are using is capturing more information into account.

Government securities in India are available in a wide range of maturities from 91 days to as long as 40 years to suit the duration of varied liability structures of various institutions.

Appendix C: Simple Example of the decomposition: 2 types of bonds example

In order to get a concise picture of our accounting decomposition, we provide a simple example consisting of 2 types of bonds with maturities in 1 and 2 time periods. Working with this example will help to see the decomposition in a much more tractable way.

Suppose we are in time period $t = 1$ and there are bonds of two types of maturities. The maturity profile of the bond is $j = 1, 2$, so we have a short-duration bond that matures in 1 time period forward and another bond that will mature in time periods.

Thus we have $t = 0, 1, 2, 3$ and $j = 1, 2$.

So the consolidated budget constraint of the government in period 1 is given as

$$\frac{B_1}{Y_1} = (r_{0,1} - \pi_{0,1} - g_{0,1})\frac{B_0}{Y_0} + \frac{def_1}{Y_1} + \frac{B_0}{Y_0} \tag{34}$$

For sake of simplicity we assume that there are only nominal zero coupon bonds and not inflation-indexed. So let $\tilde{B}_0^j$ be real values of nominal zero-coupon bonds of maturity j at time *0*.

If we denote the holding period return (HPR) between time *0 and 1* by $\tilde{r}_{0,1}^j$, then we can write the new budget constraint at time $t = 1$ as

$$\frac{\tilde{B}_1}{Y_t} = \left(\tilde{r}_{0,1}^1 \frac{\tilde{B}_0^1}{Y_0} + \tilde{r}_{0,1}^2 \frac{\tilde{B}_0^2}{Y_0}\right) - (\pi_{0,1} - g_{0,1})\frac{\tilde{B}_0}{Y_0} + \frac{def_1}{Y_1} + \frac{\tilde{B}_0}{Y_0} \tag{35}$$

Again to carry out the decomposition, we have taken the same route as before and thus we can write the total real value of outstanding debt in period 1 as

$$v_1(q_2^1 s_2^1 + q_3^1 s_3^1)$$

where s_2^1 and s_3^1 are the number of time 2 and time 3 dollars that the government has at *time 1* promised to deliver.
q_2^1 and q_3^1 are the number of time 1 dollar that it takes to buy a dollar at time 2 and time 3 respectively, akin to price of a bond and thus

$$q_2^1 = \frac{1}{1 + \rho_{11}}$$
$$q_3^1 = \frac{1}{(1 + \rho_{21})^2}$$

where ρ_{11} and ρ_{21} are the YTM at time 1 for bonds maturing with 1 and 2 periods to maturity.

v_1 is simply to convert time 1 dollars into goods, so $v_1 = 1/p_1$.

Thus now we can define the time 1 budget constraint as

$$v_1(q_2^1 s_2^1 + q_3^1 s_3^1) = v_1(q_1^1 s_1^0 + q_2^1 s_2^0) + def_1 \tag{36}$$

Note that from equation 3, we will have $q_1^1 = 1$ as the number of time 1 dollar required to buy a dollar in time 1 is simply 1 itself.

To arrive at the equation, that we have in the paper, divide by Y_1 on both sides of equation 3.

$$\frac{v_1(q_2^1 s_2^1 + q_3^1 s_3^1)}{Y_1} = \frac{v_1(q_1^1 s_1^0 + q_2^1 s_2^0)}{Y_1} + \frac{def_1}{Y_1}$$

After doing the similar manipulation like we have done in the paper, we will arrive at the following equation which is given as

$$\begin{aligned}\frac{v_1(q_2^1 s_2^1 + q_3^1 s_3^1)}{Y_1} &= \left(\frac{v_1}{v_0}\frac{q_1^1}{q_1^0}\frac{Y_0}{Y_1} - 1\right)\frac{v_0 q_1^0 s_1^0}{Y_0} + \left(\frac{v_1}{v_0}\frac{q_2^1}{q_2^0}\frac{Y_0}{Y_1} - 1\right)\frac{v_0 q_2^0 s_2^0}{Y_0} \\ &+ \frac{def_1}{Y_1} + \frac{v_0(q_1^0 s_1^0 + q_2^0 s_2^0)}{Y_0}\end{aligned} \tag{37}$$

Now to recognize the terms and see the analogy, we have

$$v_0 q_1^0 s_1^0 = \tilde{B}_0^1$$

$$v_0 q_2^0 s_2^0 = \tilde{B}_0^2$$

$$\tilde{B}_0 = \tilde{B}_0^1 + \tilde{B}_0^2$$

Also, by taking the relevant approximations,

$$\left(\frac{v_1}{v_0}\frac{q_1^1}{q_1^0}\frac{Y_0}{Y_1} - 1\right) \approx \tilde{r}_{0,1}^1 - \pi_{0,1} - g_{0,1}$$

$$\left(\frac{v_1}{v_0}\frac{q_2^1}{q_2^0}\frac{Y_0}{Y_1} - 1\right) \approx \tilde{r}_{0,1}^2 - \pi_{0,1} - g_{0,1}$$

Since by definition, $\tilde{r}_{0,1}^1$ is the net nominal holding period return of a 1 period to maturity bond between periods 0 and 1, hence it is actually nothing but $\tilde{r}_{0,1}^1 = \rho_{10}$, where ρ_{10} is the YTM of a one period to maturity bond at time 0.

In order to decompose the term premia (excess return) component from the nominal returns, we need to keep in mind the fact that by definition, term premium or excess return is the excess holding period return on holding a zero coupon bond for different maturities. To be more precise, it is the excess return of a long bond over a short bond.

In our case, we will have

$$\tilde{r}_{0,1}^1 = \rho_{10}$$

$$\tilde{r}^2_{0,1} = \rho_{10} + \tilde{rx}^2_{0,1} \tag{38}$$

where $\tilde{rx}^2_{0,1}$ is the excess return of holding a 2 period to maturity bond from periods 0 to 1.

Excess returns or Term Premium is in general the compensation demanded by investors for holding duration risk. So it's the compensation demanded for holding the longer duration asset as against rolling the investment with short-term instruments.

Note that $\tilde{r}^1_{0,1}$ will have no excess return as its the HPR of a 1 period to maturity bond between time 0 and 1.

Thus, from equation 6 we have

$$\tilde{rx}^2_{0,1} = \tilde{r}^2_{0,1} - \rho_{10}$$

As can be seen from this equation that the excess returns or term premium can be negative or positive depending upon the preference of the investors and other market information like monetary policy, inflation expectations, macro scenario-like any shock or the supply side dynamics of the bond market to name some.
In principle the excess returns or term premium can be negative as well if suppose the investor prefer to secure a fixed return for long period (say for institutional pension funds), then the investor may be willing to accept a negative term premium than taking over the rolling over risk.

Now plugging all these into equation 5, we will have

$$\begin{aligned}\frac{v_1(q^1_2 s^1_2 + q^1_3 s^1_3)}{Y_1} &= \left(\frac{v_1}{v_0}\frac{q^1_1}{q^0_1}\frac{Y_0}{Y_1} - 1\right)\frac{v_0 q^0_1 s^0_1}{Y_0} + \left(\frac{v_1}{v_0}\frac{q^1_2}{q^0_2}\frac{Y_0}{Y_1} - 1\right)\frac{v_0 q^0_2 s^0_2}{Y_0} \\ &\quad + \frac{def_1}{Y_1} + \frac{v_0(q^0_1 s^0_1 + q^0_2 s^0_2)}{Y_0}\end{aligned}$$

$$\begin{aligned}&\implies \frac{\tilde{B}_1}{Y_1} = \left(\tilde{r}^1_{0,1} - \pi_{0,1} - g_{0,1}\right)\frac{\tilde{B}^1_0}{Y_0} + \left(\tilde{r}^2_{0,1} - \pi_{0,1} - g_{0,1}\right)\frac{\tilde{B}^2_0}{Y_0} + \frac{def_1}{Y_1} + \frac{\tilde{B}_0}{Y_0} \\ &\implies \frac{\tilde{B}_1}{Y_1} - \frac{\tilde{B}_0}{Y_0} = (\tilde{r}^1_{0,1})\frac{\tilde{B}^1_0}{Y_0} + (\tilde{r}^2_{0,1})\frac{\tilde{B}^2_0}{Y_0} - (\pi_{0,1} + g_{0,1})\left(\frac{\tilde{B}^1_0}{Y_0} + \frac{\tilde{B}^2_0}{Y_0}\right) + \frac{def_1}{Y_1} \\ &\implies \frac{\tilde{B}_1}{Y_1} - \frac{\tilde{B}_0}{Y_0} = (\rho_{10})\frac{\tilde{B}^1_0}{Y_0} + (\rho_{10} + \tilde{rx}^2_{0,1})\frac{\tilde{B}^2_0}{Y_0} - (\pi_{0,1} + g_{0,1})\left(\frac{\tilde{B}_0}{Y_0}\right) + \frac{def_1}{Y_1} \qquad (39)\end{aligned}$$

Equation 30 is the decomposition equation, which is actually taking into account the fact that we have a term premium term explicitly floating in it.

Example of Negative Excess Returns

Now, it can be quite possible to have the term premium/ excess returns to be negative.

Recall, by definition, the excess returns in this framework is given as

$$\tilde{rx}^2_{0,1} = \tilde{r}^2_{0,1} - \rho_{10}$$

There can be two possible ways for the excess return to be negative:

1. When the short rate ,ρ_{10}, is greater than the HPR of a 2 periods to maturity bond between time 0 and 1, $\tilde{r}^2_{0,1}$.
2. The nominal returns, $\tilde{r}^2_{0,1}$, is itself negative to begin with.

Let's see an instance when the second case is possible, as we have episodes in our dataset where the nomial returns themselve become negative.

Remember that y definition

$$\tilde{r}^2_{0,1} = \frac{q^1_2}{q^0_2} - 1$$

where $q^1_2$is the number of time 1 rupee that it takes to buy a rupees at time 2 and q^0_2 is the number of time 0 rupee required to buy a rupees at time 2. Therefore

$$\begin{aligned}\tilde{r}^2_{0,1} &= \frac{q^1_2}{q^0_2} - 1 \\ &= \frac{\frac{1}{(1+\rho_{11})}}{\frac{1}{(1+\rho_{20})^2}} - 1\end{aligned} \tag{40}$$

For example take the case that

$$\rho_{11} = 0.05$$

$$\rho_{20} = 0.02$$

then our calculations will yield

$$\tilde{r}^2_{0,1} = \frac{\frac{1}{(1.05)}}{\frac{1}{(1.04)^2}} - 1$$

$$\implies \tilde{r}^2_{0,1} = \frac{0.9523}{0.9611} - 1$$

$$\implies \tilde{r}^2_{0,1} = -0.009$$

So as the YTM increases, as we can clearly see from the example, the price of the security falls and thus it can be possible to have negative returns.

Appendix D: Derivations of the Static Model Equations

Variance of Taxation

To compute $Var(\tau)$, use the financing equation of the fiscal authority

$$\underbrace{\tau}_{receipts} = \underbrace{\alpha r D + (1-\alpha)\hat{r}D + G - \pi\alpha D}_{expenditure}$$

$$\begin{aligned}
Var(\tau) =& Var(\alpha r D + (1-\alpha)\hat{r}D + G - \pi\alpha D) \\
&= Var(r)D^2\alpha^2 + Var(G) + Var(\pi)D^2\alpha^2 + 2D\alpha Cov(r,G) \\
&- 2D\alpha Cov(\pi,G) - 2D^2\alpha^2 Cov(\pi,r)
\end{aligned} \tag{41}$$

where we have made use of the fact that the long-term rate $\hat{r}$ doesn't vary on its own and depends on future short-term rates. It is like long-term rate depends on expected future short-term rates.

FOC with respect to α

FOC with respect to α will give us

$$\begin{aligned}
& rD - \hat{r}D + \frac{\lambda}{2}\left[2D^2Var(r)\alpha + 2D^2Var(\pi)\alpha + 2DCov(G,r) - 2DCov(G,\pi) - 2D^2Cov(\pi,r)2\alpha\right] \\
& - [\mathcal{P}_\alpha(r,\alpha,\pi) - pD] = 0 \\
& \implies (r-\hat{r})D + \lambda\left[D^2Var(r)\alpha + D^2Var(\pi)\alpha + DCov(G,r) - DCov(G,\pi) - 2D^2Cov(\pi,r)\alpha\right] \\
& - [\mathcal{P}_\alpha(r,\alpha,\pi) - pD] = 0 \\
& \implies (r-\hat{r})D + \lambda D^2Var(r)\alpha + \lambda D^2Var(\pi)\alpha + \lambda DCov(G,r) - \lambda DCov(G,\pi) - \lambda 2D^2Cov(\pi,r) \\
& - [\mathcal{P}_\alpha(r,\alpha,\pi) - pD] = 0 \\
& \implies \lambda D^2Var(r)\alpha + \lambda D^2Var(\pi)\alpha - 2\lambda D^2Cov(\pi,r) = (\hat{r}-r)D + [\mathcal{P}_\alpha(r,\alpha,\pi) - pD] \\
& - \lambda DCov(G,r) + \lambda DCov(G,\pi) \\
& \implies \alpha\lambda D^2[Var(r) + Var(\pi) - 2Cov(\pi,r)] = (\hat{r}-r)D + [\mathcal{P}_\alpha(r,\alpha,\pi) - pD] \\
& - \lambda DCov(G,r) + \lambda DCov(G,\pi)
\end{aligned}$$

$$\boxed{\implies \alpha = \frac{(\hat{r}-r)}{\lambda D\mathrm{A}} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D^2\mathrm{A}} - \frac{Cov(r,G)}{D\mathrm{A}} + \frac{Cov(G,\pi)}{D\mathrm{A}}} \tag{42}$$

where $\mathrm{A} = Var(r) + Var(\pi) - 2Cov(\pi,r)$

Derivation of Comparative Statics 1

We start with

$$\begin{aligned}
\alpha^* &= \frac{(\hat{r}-r)}{\lambda D \mathrm{A}} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D^2 \mathrm{A}} - \frac{Cov(r,G)}{D\mathrm{A}} + \frac{Cov(G,\pi)}{D\mathrm{A}} \\
\frac{\partial \alpha^*}{\partial D} &= \frac{-(\hat{r}-r)}{\lambda D^2 \mathrm{A}} + \frac{\left(\lambda D^2 \mathrm{A}(-p) - (\mathcal{P}_\alpha(r,\alpha,\pi) - pD)2\lambda D\mathrm{A}\right)}{(\lambda D^2\mathrm{A})^2} + \frac{Cov(r,G)}{D^2\mathrm{A}} - \frac{Cov(G,\pi)}{D^2\mathrm{A}} \\
\frac{\partial \alpha^*}{\partial D} &= \frac{-(\hat{r}-r)}{\lambda D^2 \mathrm{A}} + \frac{\lambda D\mathrm{A}\left(-pD - 2\mathcal{P}_\alpha(r,\alpha,\pi) + 2pD\right)}{(\lambda D^2\mathrm{A})^2} + \frac{Cov(r,G)}{D^2\mathrm{A}} - \frac{Cov(G,\pi)}{D^2\mathrm{A}}
\end{aligned}$$

Now from our data, we have $\frac{\partial \alpha^*}{\partial D} < 0$. Thus in our case, to get this inequality

$$\begin{aligned}
&\frac{\partial \alpha^*}{\partial D} < 0 \\
&\implies \frac{-(\hat{r}-r)}{\lambda D^2 \mathrm{A}} + \frac{\lambda D\mathrm{A}\left(-pD - 2\mathcal{P}_\alpha(r,\alpha,\pi) + 2pD\right)}{(\lambda D^2\mathrm{A})^2} + \frac{Cov(r,G)}{D^2\mathrm{A}} - \frac{Cov(G,\pi)}{D^2\mathrm{A}} < 0 \\
&\implies \frac{\lambda D\mathrm{A}\left(pD - 2\mathcal{P}_\alpha(r,\alpha,\pi)\right)}{(\lambda D^2\mathrm{A})^2} + \frac{Cov(r,G)}{D^2\mathrm{A}} < \frac{(\hat{r}-r)}{\lambda D^2 \mathrm{A}} + \frac{Cov(G,\pi)}{D^2\mathrm{A}} \\
&\implies \frac{Cov(r,G)}{D^2\mathrm{A}} < \frac{(\hat{r}-r)}{\lambda D^2 \mathrm{A}} + \frac{Cov(G,\pi)}{D^2\mathrm{A}} + \frac{(2\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{(\lambda D^3\mathrm{A})} \\
&\implies \boxed{Cov(r,G) < \frac{(\hat{r}-r)}{\lambda} + Cov(G,\pi) + \frac{(2\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{(\lambda D)}} \qquad (43)
\end{aligned}$$

Derivation for Comparative Statics 2

Variance of r

In order to find $\frac{\partial Var(r)}{\partial r}$, need to work with its distribution. Assume that r is a non-negative continuous random variable with probability density function $f(r)$ whose expectation is given as

$$\mathbf{E}(r) = \int_0^\infty r f(r) dr$$

and variance as

$$Var(r) = \mathbf{E}(r^2) - (\mathbf{E}(r))^2 = \int_0^\infty r^2 f(r) dr - \left(\int_0^\infty r f(r) dr\right)^2$$

Assume that expectation and variance $< \infty$. Then it can be shown that, for any general PDF $f(r)$

$$
\begin{aligned}
Var(r) &= \int_0^\infty r^2 f(r)dr - \left(\int_0^\infty rf(r)dr\right)^2 \\
\implies \frac{\partial Var(r)}{\partial r} &= \frac{d}{dr}\left[\int_0^\infty r^2 f(r)dr - \left(\int_0^\infty rf(r)dr\right)^2\right] \\
\implies \frac{\partial Var(r)}{\partial r} &= \frac{d}{dr}\int_0^\infty r^2 f(r)dr - \frac{d}{dr}\left(\int_0^\infty rf(r)dr\right)^2 \\
\implies \frac{\partial Var(r)}{\partial r} &= r^2 f(r) - 2\left(\int_0^\infty rf(r)dr\right)\left(\frac{d}{dr}\int_0^\infty rf(r)dr\right) \\
\implies \frac{\partial Var(r)}{\partial r} &= r^2 f(r) - 2\left(\int_0^\infty rf(r)dr\right)(rf(r)) \\
\implies & \boxed{\frac{\partial Var(r)}{\partial r} = r^2 f(r) - 2\mathbf{E}(r)\,(rf(r)) = rf(r)(r - 2\mathbf{E}(r))}
\end{aligned} \tag{44}
$$

$$
\frac{\partial Var(r)}{\partial r} = rf(r)(r - 2\mathbf{E}(r))
$$

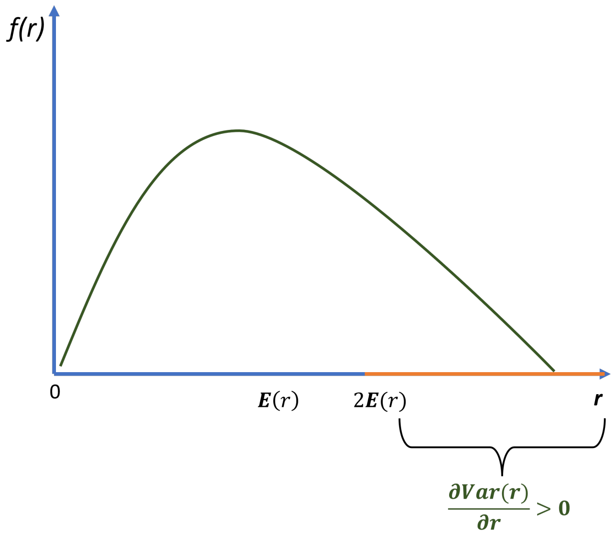

Figure 16: $f(r)$ vs r

Comparative Statics 2 Derivation

Again we start with the condition of optimal α

$$
\begin{aligned}
\alpha^* &= \frac{(\hat{r} - r)}{\lambda D\mathrm{A}} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D^2\mathrm{A}} - \frac{Cov(r,G)}{D\mathrm{A}} + \frac{Cov(G,\pi)}{D\mathrm{A}} \\
&= \frac{1}{D\mathrm{A}}\left[\frac{(\hat{r} - r)}{\lambda} + \frac{(\mathcal{P}_\alpha(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right]
\end{aligned} \tag{45}
$$

Taking the derivative of equation (11), we have

$$\begin{aligned}\frac{\partial \alpha^*}{\partial r} =& \frac{1}{D\mathrm{A}}\left[\frac{-1}{\lambda} + \frac{\mathcal{P}_{\alpha r}(r,\alpha,\pi)}{\lambda D} - \frac{\partial Cov(r,G)}{\partial r}\right] \\ & - \frac{1}{D\mathrm{A}^2}\frac{\partial \mathrm{A}}{\partial r}\left[\frac{(\hat{r}-r)}{\lambda} + \frac{(\mathcal{P}_{\alpha}(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right] \end{aligned} \tag{46}$$

Given that we want $\frac{\partial \alpha^*}{\partial r} > 0$ (negative relation between the short-term nominal rate and the average maturity of debt), which means

$$\begin{aligned}&\frac{\partial \alpha^*}{\partial r} > 0 \\ &\implies \frac{1}{D\mathrm{A}}\left[\frac{-1}{\lambda} + \frac{\mathcal{P}_{\alpha r}(r,\alpha,\pi)}{\lambda D} - \frac{\partial Cov(r,G)}{\partial r}\right] \\ &> \frac{1}{D\mathrm{A}^2}\frac{\partial \mathrm{A}}{\partial r}\left[\frac{(\hat{r}-r)}{\lambda} + \frac{(\mathcal{P}_{\alpha}(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right] \\ &\implies \mathrm{A}\left[\frac{-1}{\lambda} + \frac{\mathcal{P}_{\alpha r}(r,\alpha,\pi)}{\lambda D} - \frac{\partial Cov(r,G)}{\partial r}\right] \\ &> \frac{\partial \mathrm{A}}{\partial r}\left[\frac{(\hat{r}-r)}{\lambda} + \frac{(\mathcal{P}_{\alpha}(r,\alpha,\pi) - pD)}{\lambda D} - Cov(r,G) + Cov(G,\pi)\right] \end{aligned} \tag{47}$$

Proof of $\frac{\partial A}{\partial r} > 0$

Start with

$$\mathrm{A} = Var(r) + Var(\pi) - 2Cov(\pi, r)$$

Now

$$\frac{\partial A}{\partial r} = \frac{\partial Var(r)}{\partial r} + \frac{\partial Var(\pi)}{\partial r} - \frac{2Cov(\pi,r)}{\partial r}$$

From Taylor rule

$$r = \phi(\pi - \pi^*)$$

$$\implies Var(r) = \phi^2 Var(\pi)$$

$$\implies \frac{\partial Var(r)}{\partial r} = \phi^2\frac{\partial Var(\pi)}{\partial r}$$

Since we are considering $\frac{\partial Var(r)}{\partial r} > 0$, thus

$$\frac{\partial Var(\pi)}{\partial r} > 0$$

Again, from the Taylor rule

$$r = \phi(\pi - \pi^*)$$

$$\implies Cov(\pi, r) = \phi Var(\pi)$$
$$\implies \frac{2Cov(\pi, r)}{\partial r} = \phi \frac{\partial Var(\pi)}{\partial r}$$

Plug this in the equation for $\frac{\partial A}{\partial r}$

$$\frac{\partial A}{\partial r} = \phi^2 \frac{\partial Var(\pi)}{\partial r} + \frac{\partial Var(\pi)}{\partial r} - 2\phi \frac{\partial Var(\pi)}{\partial r}$$
$$\implies \frac{\partial A}{\partial r} = \frac{\partial Var(\pi)}{\partial r}(\phi - 1)^2$$

Since $\frac{\partial Var(r)}{\partial r} > 0$, thus

$$\frac{\partial A}{\partial r} > 0$$

Proof of $\frac{\partial Cov(r, G)}{\partial r} < 0$

Start with

$$\begin{aligned} Var(\tau) =& Var(\alpha r D + (1 - \alpha)\hat{r} D + G - \pi \alpha D) \\ &= Var(r)D^2\alpha^2 + Var(G) + Var(\pi)D^2\alpha^2 + 2D\alpha Cov(r, G) \\ &- 2D\alpha Cov(\pi, G) - 2D^2\alpha^2 Cov(\pi, r) \end{aligned} \tag{48}$$

$$\implies \frac{\partial Cov(r, G)}{\partial r} = \frac{D^2\alpha^2}{2\alpha D}\left[- \frac{\partial Var(r)}{\partial r} - \frac{\partial Var(\pi)}{\partial r} + 2\frac{\partial Cov(r, \pi)}{\partial r} \right] \tag{49}$$

Using the fact that

$$\frac{2Cov(\pi, r)}{\partial r} = \phi \frac{\partial Var(\pi)}{\partial r}$$
$$\frac{\partial Var(r)}{\partial r} = \phi^2 \frac{\partial Var(\pi)}{\partial r}$$

we have

$$\frac{\partial Cov(r, G)}{\partial r} = -\frac{D\alpha}{2} \frac{\partial Var(\pi)}{\partial r}(\phi - 1)^2 < 0$$

last step follows because $\frac{\partial Var(r)}{\partial r} > 0$

Appendix E: Data

Steps of preparing the dataset for decomposition

In this section, we provide the complete steps as to how we assembled the data and the subsequent process to make it usable for the accounting decomposition.

- We assembled the data for all the Central government securities issued by the government from 1999 on wards from the Status Paper of government debt, issued by the Ministry of Finance.
- The RBI on the behalf of the Ministry of Finance resorts to issue the securities and act as the debt manager for the government, the next section in this appendix gives some details as to how does RBI carry out the task.
- Since a coupon bond is a stream of promised coupons plus an ultimate principal payment. We regard such a bond as a bundle of zero-coupon bonds of different maturities and price it by unbundling it into the underlying component zero-coupon bonds, one for each date at which a coupon or principal is due, valuing each promised payment separately.
- In this way we can get the **C** matrix, which gives the coupon payments for all the securities over all the years for all the maturities and the **P** matrix, which is the principal matrix that gives the principal payments for all the years over the whole maturity horizon.
- So in a way we have stripped the coupons from each bond and price a bond as a weighted sum of zero-coupon bonds of maturities.
- By adding these two matrices, namely the P matrix and the C matrix, we can get the total payments outstanding for the central government as a weighted sum, with weights being given by the various maturity tranches.
- From the yield to maturity (YTM) data for the Subsidiary General Ledger (SGL) transactions in government dated securities for various maturities, we obtained the "price" of each security, which in our description is defined as the number of time t rupees that it takes to buy a rupee at time t + j. In this respect note that all the securities under our consideration are rupee denominated securities.
- We also calculate the value of currency measured in goods per rupee as the inverse of the price in the base year(this becomes our v_t.
- By multiplying q^t_{t+j} with v_t and s^t_{t+j}, we get the real value of the marketable bond in year t. Then by summing them over all the maturities, we get the total real value of government debt outstanding in period t.

- We combined this data with other variables used for decomposition like GDP that is obtained from the Economic Survey, CPI inflation which is obtained from OECD and the data for primary deficit/surplus is obtained from RBI.

- The schematic chart below gives a snapshot as to how the real value of the marketable debt is calculated.

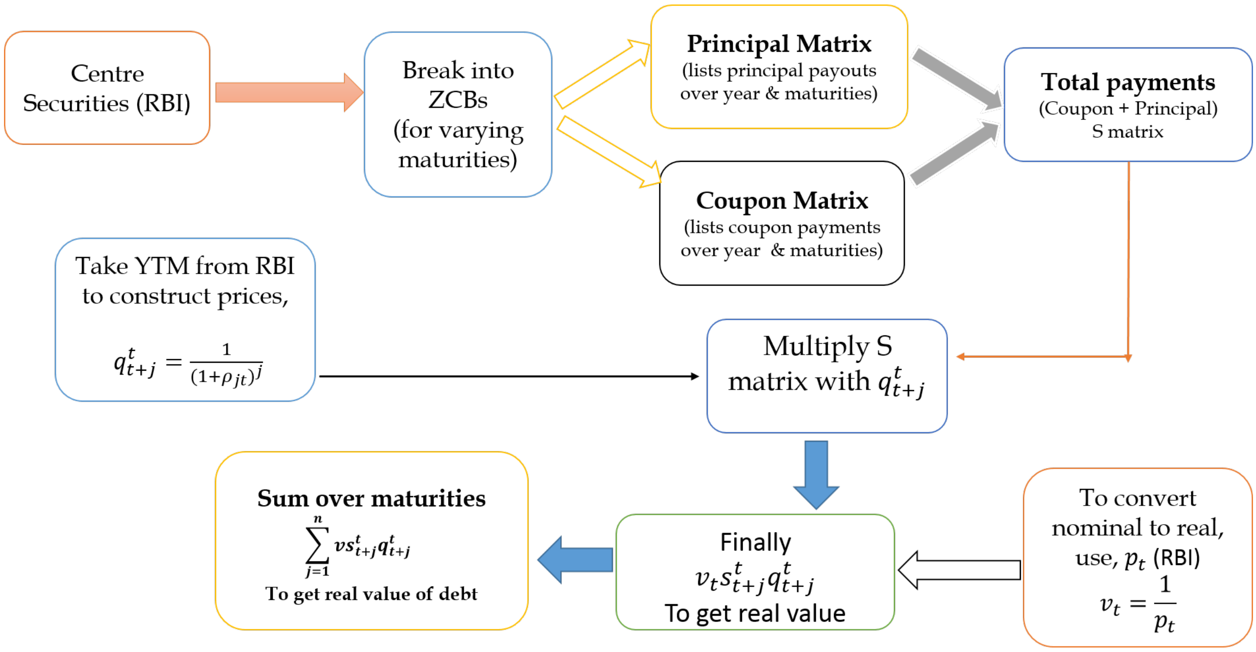

Figure 17: Schematic Flow Chart showing calculation of real value of government debt

How does the RBI issues securities?

Here we look at how the Reserve Bank of India (RBI) issues the central government securities[27].

The RBI acts as the banker and the debt manager to the government. A Government Security (G-Sec) is a tradable instrument issued by the Central Government or the State Governments acknowledging the obligation of the government's debt.

In India, the Central Government issues both, treasury bills and bonds or dated securities while the State Governments issue only bonds or dated securities, which are called the State Development Loans (SDLs).

The *Public Debt Office (PDO)* of the Reserve Bank of India acts as the registry / depository of G-Secs and deals with the issue, interest payment and repayment of principal at maturity. Most of the *dated securities* are **fixed coupon** securities.

Types of bonds issued

[27] Details can be found here

- Most Government bonds in India are issued as fixed rate bonds.
- Floating rate bonds (FRBs) were first issued in September 1995 in India and have a variable coupon and can carry the coupon, which will have a base rate plus a fixed spread, to be decided by way of auction mechanism.
- Government had last issued a zero coupon bond in 1996.
- Inflation Indexed Bonds (IIBs) - IIBs are bonds wherein both coupon flows and Principal amounts are protected against inflation.
- STRIPS: they are essentially zero coupon bonds and they are created out of existing securities only and unlike other securities, are not issued through auctions.

Besides banks, insurance companies and other large investors, smaller investors like Co-operative banks, Regional Rural Banks, Provident Funds are also required to statutory hold G-Secs.

How are G-Secs issued

- G-Secs are issued through auctions conducted by RBI on its electronic platform.
- Participants include Commercial banks, scheduled UCBs, Primary Dealers, insurance companies and provident funds, who maintain funds account (current account) and securities accounts (Subsidiary General Ledger (SGL) account) with RBI.
- All non members including non-scheduled UCBs can participate in the primary auction through scheduled commercial banks or PDs.
- The RBI, in consultation with the Government of India, issues an indicative half-yearly auction calendar which contains information.
- Auction for *dated securitie*s is conducted on Friday for settlement on T+1 basis (i.e. securities are issued on next working day i.e. Monday).
- The Reserve Bank of India conducts auctions usually every Wednesday to issue T-bills of 91day, 182 day and 364 day tenors. Settlement for the *T-bills* auctioned is made on T+1 day.
- An auction may either be yield based or price based
  - A **yield-based auction** is generally conducted when a **new** G-Sec is issued. Investors bid in yield terms up to two decimal places. Bids are arranged in ascending order and the cut-off yield is arrived at the yield corresponding to the notified amount of the auction and the cut-off yield is then fixed as the coupon rate for the security.Bids which are higher than the cut-off yield are *rejected*.

- A **price based auction** is conducted when Government of India **re-issues securities** which have already been issued earlier. Bidders quote in terms of price per '100 of face value of the security. Bids are arranged in *descending order* of price offered and the successful bidders are those who have bid at or above the cut-off price.
- Depending upon the method of allocation to successful bidders, auction may be conducted on Uniform Price basis or Multiple Price basis.
- In a **competitive bidding**, an investor bids at a specific price / yield and is allotted securities if the price / yield quoted is within the cut-off price / yield and are undertaken by well-informed institutional investors such as banks, financial institutions, PDs, mutual funds, and insurance companies.

How does one get information about the price of a G-Sec?

The return on a security is a combination of two elements (i) coupon income and (ii) the gain / loss on the security due to price changes.

Information on traded prices of securities is available on the RBI website here and also in the FBIL website.

The Clearing Corporation of India Limited (CCIL) is the clearing agency for G-Secs. In effect, during settlement, the CCP becomes the seller to the buyer and buyer to the seller of the actual transaction.CCIL also guarantees settlement of all trades in G-Secs.