

Multinational Firms, Trade, and the Trade-Comovement Puzzle

Gautham Udupa

CAFRAL

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Motivation

Empirical research: More trade between countries associated with increase in business cycle correlation (comovement)

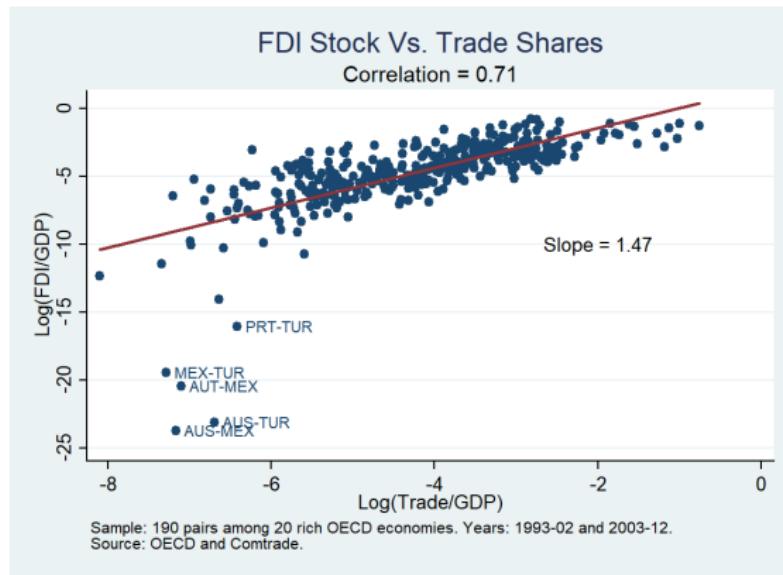
Specification

- Robust in a cross-section of country-pairs, as well as in a panel (using time variation)

Puzzle: Simulations of benchmark open economy macro model can not quantitatively match empirical research

- Simulation generates 10% of the magnitude of observed trade-comovement slope
- Model: Backus, Kehoe, and Kydland (1994)

Motivation - Bilateral Trade and FDI Stock



Countries that trade more also have high FDI linkages

What I do

- 1 Conduct trade-comovement regression with FDI as an additional variable
 - FDI is significant
- 2 Develop dynamic model of trade and FDI with heterogeneous firms
- 3 Calibrate and simulate the model
 - Run simulated data regression
 - Model can generate empirical patterns

Outline

- 1 Existing literature
- 2 Empirical analysis
- 3 Model
- 4 Calibration and simulation
- 5 Results and mechanisms

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Existing Literature

Trade and comovement

Frankel and Rose (1998), Imbs (2004), Kose and Yi (2006), Clark and van Wincoop (2001), Baxter and Kouparitsas (2005), Inklaar et al.(2008), Ng (2010), Liao and Santacreu (2015), [Soyres \(2017\)](#)

Multinationals and comovement

Cravino and Levchenko (2017), di Giovanni et al. (2017), Alviarez, Cravino, and Levchenko (2017)

Multinationals, trade, and comovement

Jansen and Stokman (2014), Kleinert, Martin, and Toubal (2014)

Theory

Kose and Yi (2006), Johnson (2014), Liao and Santacreu (2015), [Soyres \(2017\)](#)

- 1 Existing literature
- 2 Empirical analysis**
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Empirical Analysis - Specification

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

$\text{Corr}_\tau(Y_{it}, Y_{jt})$ is correlation of residuals from HP filtered quarterly real GDP during time period τ .

$\text{Trade}_{ij\tau} = \text{mean}_{t \in \tau} \left[\max \left\{ \frac{X_{ijt} + M_{ijt}}{GDP_{it}}, \frac{X_{ijt} + M_{ijt}}{GDP_{jt}} \right\} \right]$ is a measure of bilateral trade intensity during time period τ .

$\text{FDI}_{ij\tau} = \text{mean}_{t \in \tau} \left[\max \left\{ \frac{\text{In stock}_{ijt} + \text{Out stock}_{ijt}}{GDP_{it}}, \frac{\text{In stock}_{ijt} + \text{Out stock}_{ijt}}{GDP_{jt}} \right\} \right]$ is a measure of bilateral FDI stock intensity during time period τ .

δ_{ij} is country-pair fixed effects.

Time periods: (1) 1993-2002 (2) 2003-2012.

Data Sources

- Quarterly real GDP from OECD
- Annual bilateral trade from UN COMTRADE database
- Annual bilateral FDI stocks (1993-2012) from OECD

Countries: Australia, Austria, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States.

Empirical Analysis - Results

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

Table: Results - HP filtered GDP

| | (1) | (2) | (3) |
|-----------------|------------------|-------------------|-------------------|
| Log(Trade) | 0.32** (0.10) | | 0.18 (0.10) |
| Log(FDI) | | 0.07*** (0.02) | 0.06*** (0.02) |
| Country-pair FE | Yes | Yes | Yes |
| Observations | 364 | 364 | 364 |
| R ² | 0.47 | 0.51 | 0.52 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing FDI values within any 10-year time period.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Summary Stats

BK filter

Growth rate

Sum GDP

Product GDP

No FE

Time FE

Simulated data results

- 1 Existing literature
- 2 Empirical analysis
- 3 Model**
- 4 Calibration and simulation
- 5 Results and mechanisms

Model - Framework

- Two symmetric countries
- Endogenous number of firms which differ in productivity
- Monopolistic competition

In deterministic steady state: Helpman, Melitz, and Yeaple (2004)
plus,

- Aggregate TFP shock
- Capital accumulation
- Endogenous labor

Model - Households

- International financial autarky

Household problem ($i = \text{country index}$):

$$U_{i,0} = \max_{C_{it}, L_{it}, K_{i,t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{it}) - \psi \frac{L_{it}^{1+\nu}}{1+\nu} \right]$$

Such that

$$C_{it} + X_{it} = w_{it} L_{it} + r_{it} K_{it} \quad \forall t$$

Firms - Final Good Producer

- Non-tradeable consumption/investment good
- Perfect competition

Production Function

$$Y_{i,t} = \left[\int_{\omega \in \Omega_{it}} y_{i,t}^{\frac{\sigma-1}{\sigma}}(\omega) d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

Demand for variety

$$y^d(\omega) = \left[\frac{p(\omega)}{P} \right]^{-\sigma} Y$$

Aggregate Price

$$P_{i,t} = \left[\int_{\omega \in \Omega_{it}} p_{i,t}^{1-\sigma}(\omega) d\omega \right]^{\frac{1}{1-\sigma}}$$

Model - Intermediate Goods Producers

- Infinite potential firms
- Monopolistic competition
- Firms differ in productivities, produce unique variety
- Firms live for one period

Timeline

Model - Intermediate Goods Producers

Production function

$$Y_{ik,t}(\varphi) = \frac{Z_{k,t}}{h_{ik}} \varphi K_{k,t}^\alpha(\varphi) L_{k,t}^{1-\alpha}(\varphi)$$

Where,

i, k, and t are firm's home, host countries and time indexes respectively.

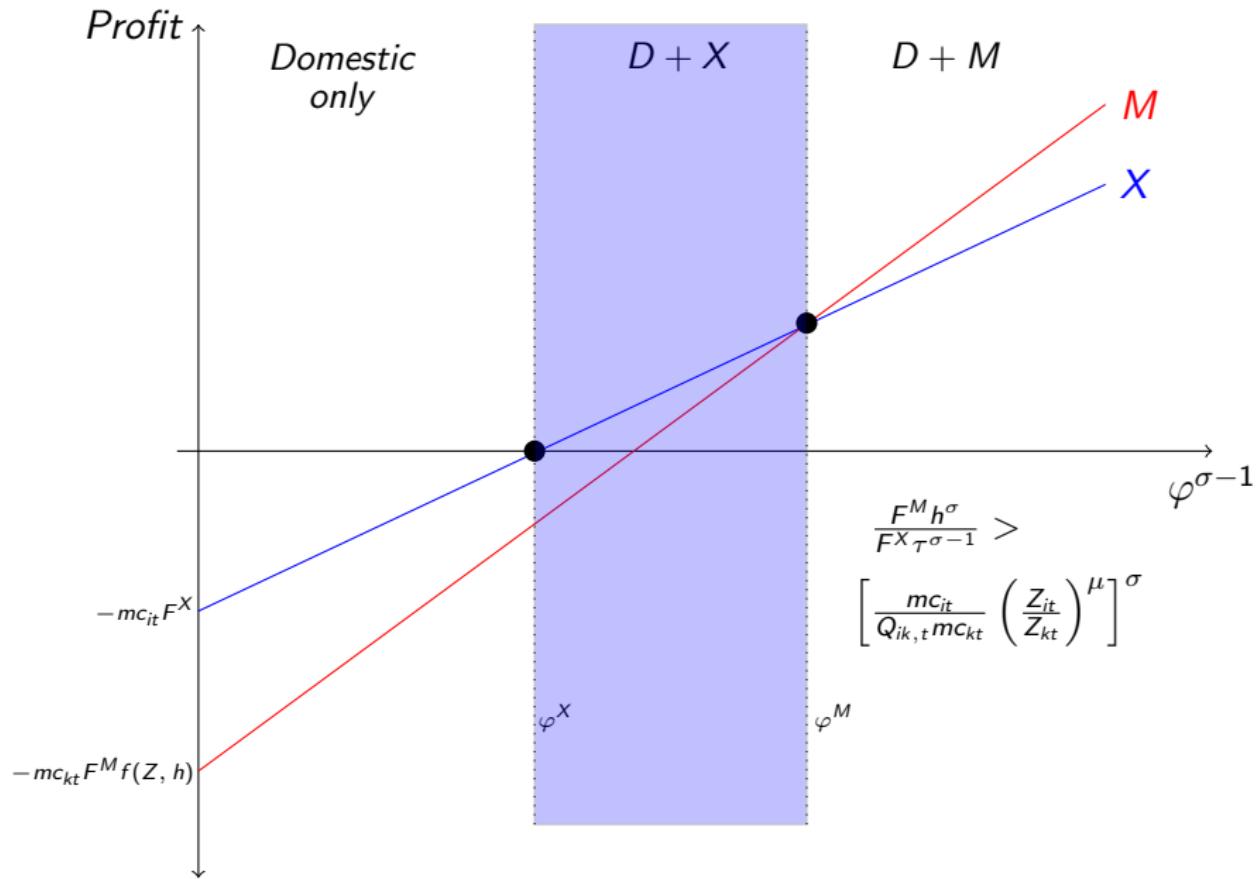
Costs:

- Export: fixed cost F^X ; iceberg cost τ
- FDI: fixed cost F^M ; tech. transfer cost h

Assumption: Exporting fixed cost is lower, but variable cost is higher.

Eq'm conditions

Static Equilibrium - Firm Profits and Choices



Model Dynamics

- 1 Aggregate TFP shocks generate business cycles in each country
 - Shocks are persistent
- 2 Firm and household choices generate additional comovement.
 - Firms: extensive and intensive margin variations among exporters and affiliates.
 - Households: demand spillover (consumption and investment)

Model Dynamics

- 1 Aggregate TFP shocks generate business cycles in each country
 - Shocks are persistent
- 2 Firm and household choices generate additional comovement.
 - Firms: extensive and intensive margin variations among exporters and affiliates.
 - Households: demand spillover (consumption and investment)

Model - Review

- Model of both trade and FDI
- Heterogeneous firms that differ in their productivity
- Entry/exit of firms into exporting and affiliate activities
- Comovement is a result of firm and household choices

- 1 Existing literature
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Calibration and Simulation - Strategy

Approach = Liao and Santacreu (2015)

Step 1: Calibrate to match intensive and extensive margins of trade and FDI for USA-Rest of the World (RoW).

Step 2: Vary only two parameters, h and τ (variable costs), to generate variation in trade and FDI intensive margins.

- X variables in the regression
- Variation in h and τ calibrated to match empirical trade-FDI slope
- Strategy designed to understand key channels of comovement in a model with FDI

Calibration - Step 1

Step 1: Calibrate to USA-Rest of the World (RoW) pair.

- RoW - Countries other than the United States in the empirical sample.
- One period = one year
- Firms = establishments in the data
- 15 parameters in total
- 4 calibrated parameters = fixed and iceberg costs (F^X , F^M , τ , and h)
 - Targets = extensive and intensive margins of trade and FDI

Baseline Calibration

| Parameter | Description | Value |
|-------------------|----------------------------|--------------|
| β | Time preference | 0.96 |
| ν | Inverse Frisch-elasticity | 0.67 |
| δ | Depreciation | 0.1 |
| σ | Elasticity of substitution | 3.5 |
| α | Capital share | 0.36 |
| ρ | AR1 parameter | 0.9 |
| σ_ϵ | AR1 shock SE | 0.02 |
| F^E | Sunk entry cost | 1 |
| γ | Pareto shape parameter | $\sigma + 2$ |
| ψ | Leisure preference | 6.7 |
| F^X | Export fixed cost | 0.18 |
| F^M | Multinational fixed cost | 0.20 |
| τ | Trade iceberg cost | 1.5 |
| h | Multinational iceberg cost | 1.4 |

Trade share = 26.4% Exporters = 34.4%
FDI share = 19.1% Affiliates = 2.7%

Simulation - Step 2

- **Step 2:** Simulate two-country model repeatedly
 - Goal: to generate variation in intensive margin of trade and FDI.
 - Vary h and τ - the iceberg costs of FDI and trade
 - Discipline the model to match empirical FDI-trade slope
- Simulation Steps
 - Result: 200 “artificial country-pairs” with data for comovement, trade, and FDI.
- Run regression akin to empirical specification with 200 observations

Simulations Vs. Data

- 1 Existing literature
- 2 Empirical analysis
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Simulation - Results

Table: Quantitative Exercises - Results

| | (1) | (2) | (3) |
|-----------------|-------------------|-------------------|-------------------|
| β_{trade} | 0.33*** (0.05) | | 0.07 (0.05) |
| β_{FDI} | | 0.08*** (0.01) | 0.07*** (0.01) |

Results from estimating trade-comovement regression with simulated data for 200 artificial country-pairs.

Intuition - Mechanisms

1 Extensive margin (EM) adjustment among exporters and affiliates

Impulse response

- Positive aggregate shock in Home \Rightarrow more Home-owned affiliates abroad
- Positive aggregate shock in Home \Rightarrow more Home exporters
- Quantitatively, EM adjustment among affiliates is more important.

2 Demand spillover

Vary τ

Vary h

EM in the data

Intuition - Mechanisms

1 Extensive margin (EM) adjustment among exporters and affiliates

Impulse response

- Positive aggregate shock in Home \Rightarrow more Home-owned affiliates abroad
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- Quantitatively, EM adjustment among affiliates is more important.

2 Demand spillover

Vary τ

Vary h

EM in the data

Conclusions

- Empirical trade-comovement slope falls when FDI is included
- Dynamic heterogeneous firms model with both trade and FDI generates observed patterns
 - EM adjustment among affiliates is quantitatively more important than EM adjustment among exporters

Trade-Comovement Specification

Cross sectional regression:

$$\text{Corr}(gdp_i, gdp_j) = \alpha + \beta \log \left(\frac{X_{ij} + M_{ij}}{GDP_i + GDP_j} \right) + \epsilon_{ij}$$

gdp_i : Cyclical component of real GDP in country i

X_{ij} : Exports, i to j

X_{ij} : Imports, j to i

GDP_i : Nominal GDP in country i

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Summary Statistics

Table: Summary Statistics

| Variable | Mean | Std. Dev. | Min. | Max. | N |
|-----------------|-------------|------------------|-------------|-------------|----------|
| HP correlation | 0.576 | 0.295 | -0.31 | 0.950 | 364 |
| BK correlation | 0.607 | 0.346 | -0.58 | 0.99 | 364 |
| GR correlation | 0.564 | 0.289 | -0.32 | 0.950 | 364 |
| FDI share | 0.039 | 0.069 | 0 | 0.461 | 364 |
| Trade share | 0.032 | 0.053 | 0 | 0.471 | 364 |

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Results Using BK Filtered GDP

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

Table: Results - BK filtered GDP

| | (1) | (2) | (3) |
|-----------------|------------------|------------------|-----------------|
| Log(Trade) | 0.36** (0.12) | | 0.27* (0.13) |
| Log(FDI) | | 0.05** (0.02) | 0.04* (0.02) |
| Country-pair FE | Yes | Yes | Yes |
| Observations | 364 | 364 | 364 |
| R ² | 0.463 | 0.462 | 0.475 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing
FDI values within any 10-year time period.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Results Using Growth Rates

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

Table: Results - Correlation of Growth Rates

| | (1) | (2) | (3) |
|-----------------|--------|-------------------|-------------------|
| Log(Trade) | 0.25* | | 0.12 |
| | (0.10) | | (0.10) |
| Log(FDI) | | 0.07*** (0.01) | 0.06*** (0.02) |
| Country-pair FE | Yes | Yes | Yes |
| Observations | 364 | 364 | 364 |
| R ² | 0.483 | 0.518 | 0.522 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing
FDI values within any 10-year time period.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Robustness

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

Table: Results - HP filtered GDP

| | (1) | (2) | (3) |
|-----------------|-------------------|-------------------|-------------------|
| Log(Trade) | 0.89*** (0.08) | | 0.79*** (0.09) |
| Log(FDI) | | 0.09*** (0.01) | 0.04** (0.01) |
| Country-pair FE | Yes | Yes | Yes |
| Observations | 364 | 364 | 364 |
| R ² | 0.657 | 0.531 | 0.672 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing FDI values within any 10-year time period.

Trade and FDI divided by sum of GDP.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Robustness

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau}$$

Table: Results - HP filtered GDP

| | (1) | (2) | (3) |
|-----------------|-------------------|-------------------|-------------------|
| Log(Trade) | 0.75*** (0.10) | | 0.62*** (0.11) |
| Log(FDI) | | 0.08*** (0.01) | 0.05** (0.01) |
| Country-pair FE | Yes | Yes | Yes |
| Observations | 364 | 364 | 364 |
| R ² | 0.575 | 0.520 | 0.598 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing

FDI values within any 10-year time period.

Trade and FDI divided by square of product of GDP.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Robustness

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \epsilon_{ij\tau}$$

Table: Results - No Fixed Effects

| | (1) | (2) | (3) |
|----------------|-------------------|-------------------|-------------------|
| Log(Trade) | 0.07*** (0.01) | | 0.02 (0.02) |
| Log(FDI) | | 0.04*** (0.01) | 0.03*** (0.01) |
| Observations | 364 | 364 | 364 |
| R ² | 0.087 | 0.133 | 0.136 |

Standard errors in parentheses

20 countries, 2 time periods and up to 9 missing FDI values within any 10-year time period.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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Robustness

$$\text{Corr}_\tau(Y_{it}, Y_{jt}) = \alpha + \beta \log(\text{Trade}_{ij\tau}) + \gamma \log(\text{FDI}_{ij\tau}) + \delta_{ij} + \delta_\tau \epsilon_{ij\tau}$$

Table: Results - HP filtered GDP

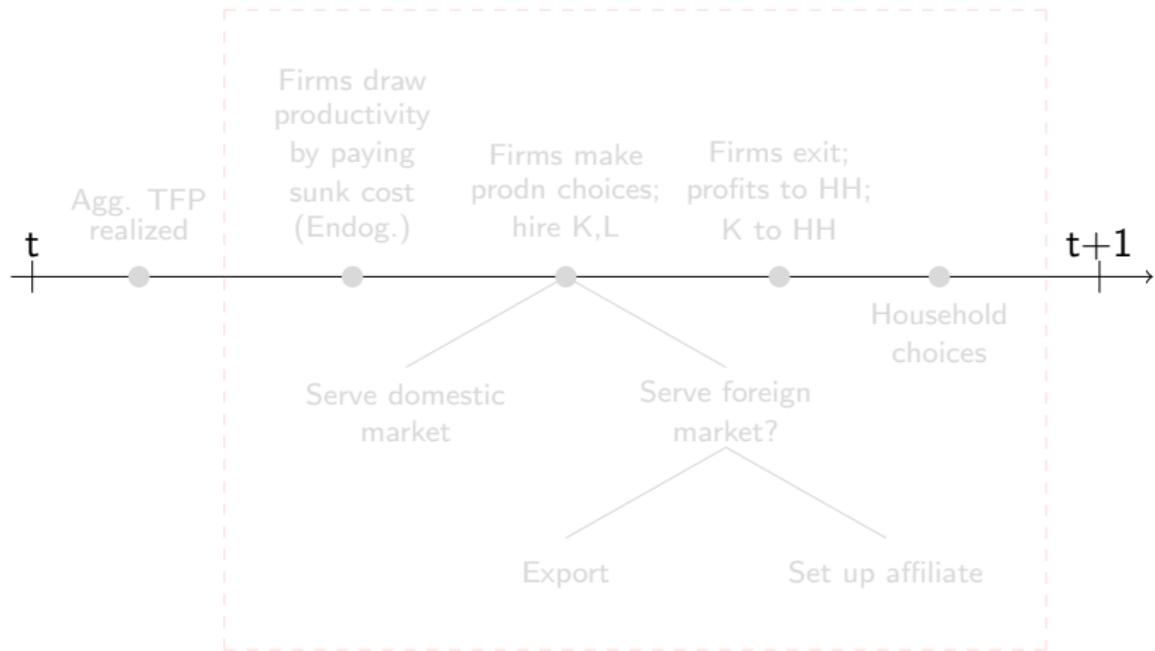
| | (1) | (2) | (3) |
|-----------------|--------|--------|--------|
| Log(Trade) | 0.14* | | 0.09 |
| | (0.06) | | (0.07) |
| Log(FDI) | | 0.05** | 0.04* |
| | | (0.02) | (0.02) |
| Country-pair FE | Yes | Yes | Yes |
| Time FE | Yes | Yes | Yes |
| Observations | 504 | 504 | 504 |
| R ² | 0.729 | 0.731 | 0.732 |

Standard errors in parentheses

20 countries, 3 time periods and up to 6 missing FDI values within any 7-year time period.

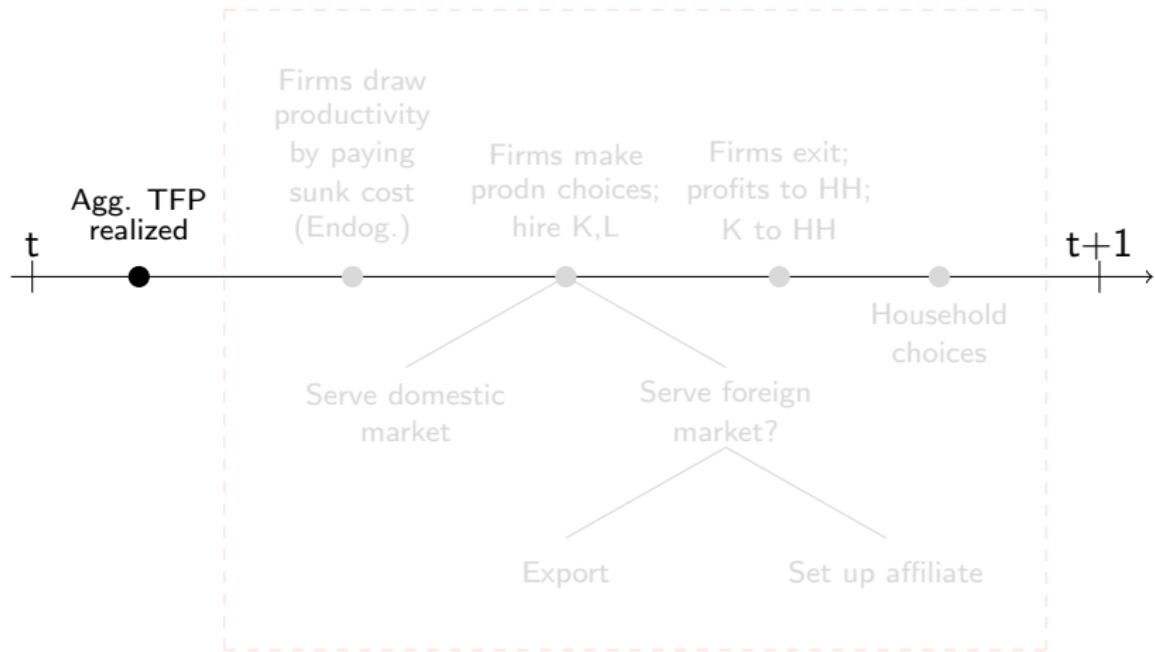
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Intermediate Goods Producers - Timeline



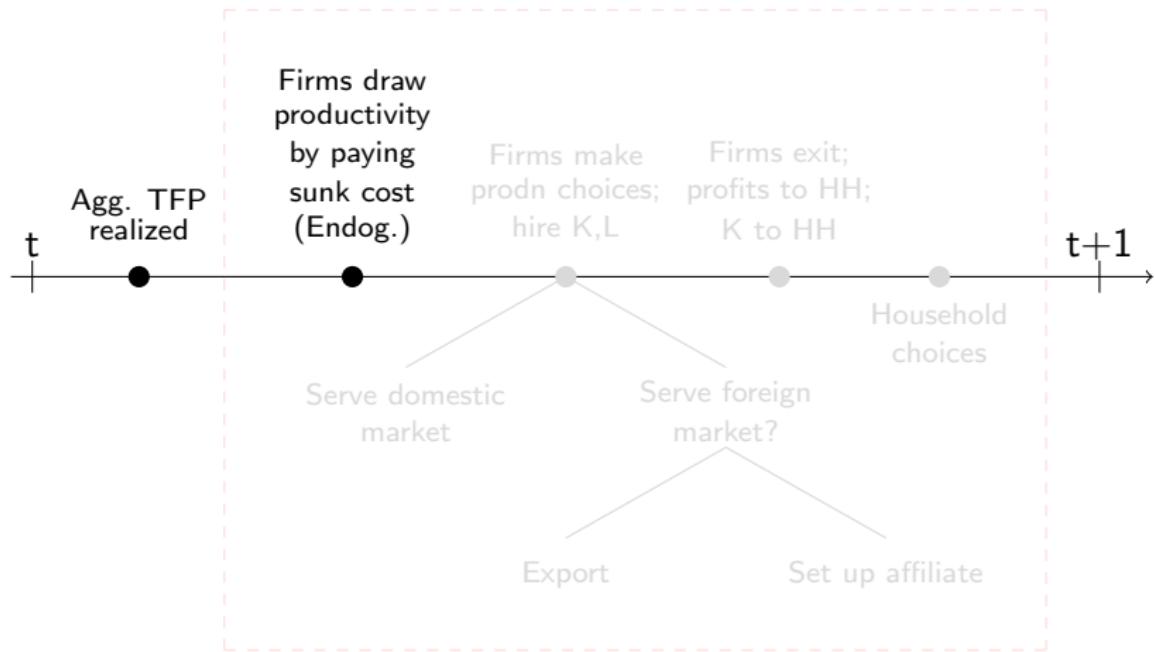
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Intermediate Goods Producers - Timeline



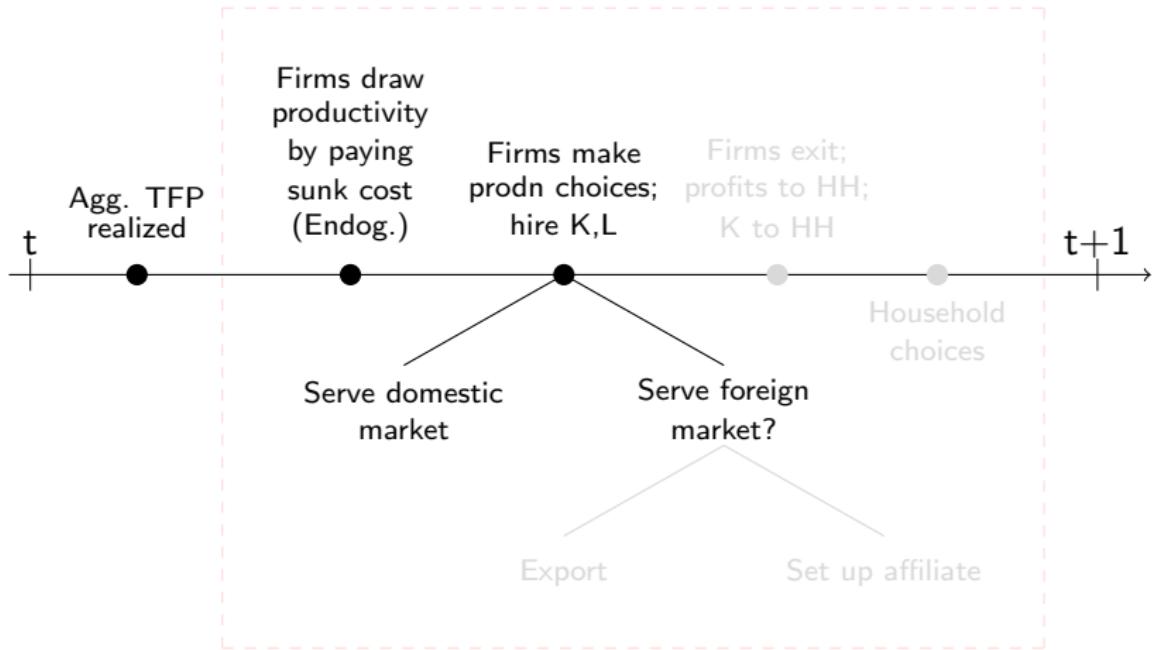
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Intermediate Goods Producers - Timeline



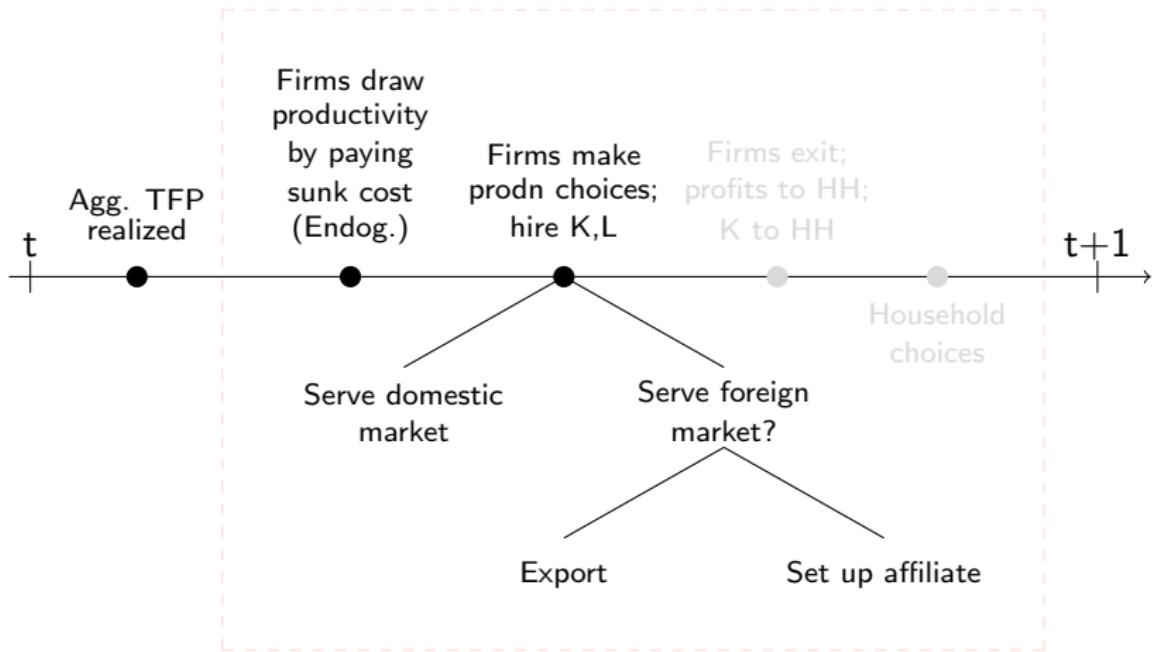
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Intermediate Goods Producers - Timeline



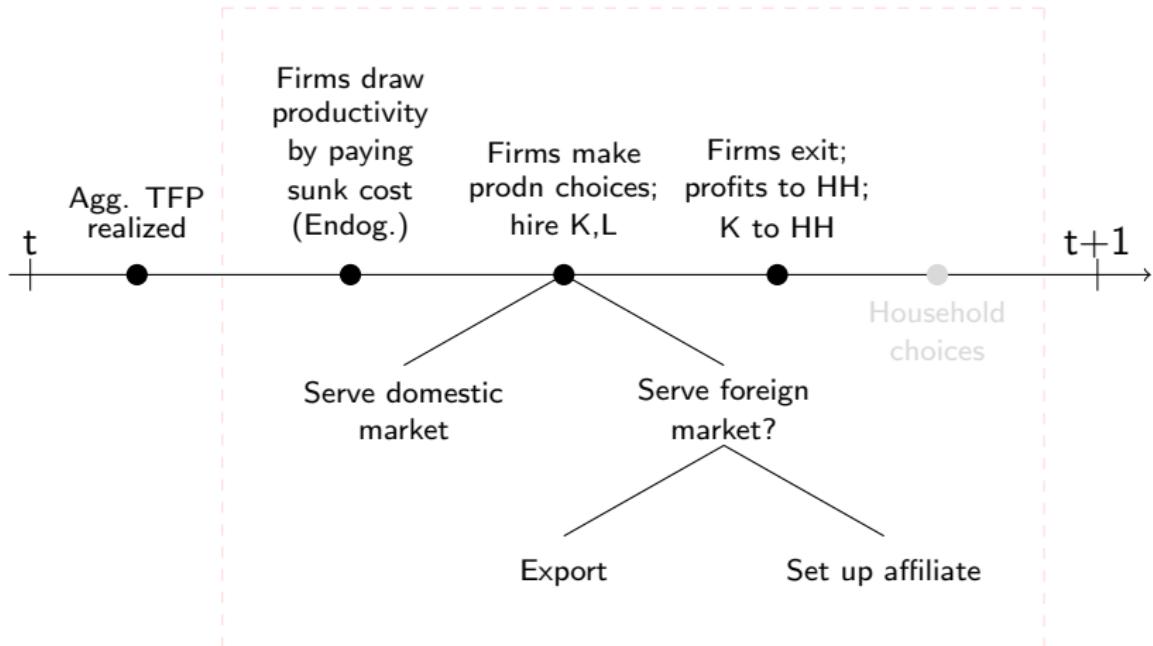
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Intermediate Goods Producers - Timeline



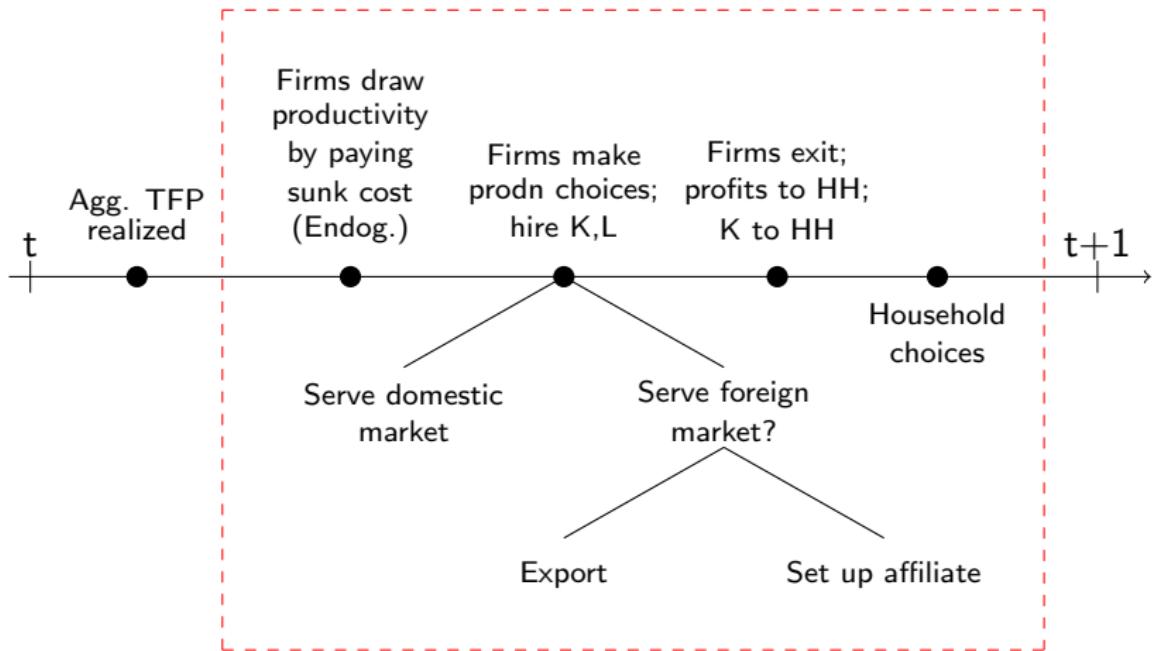
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Intermediate Goods Producers - Timeline



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Intermediate Goods Producers - Timeline



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Model- Equilibrium Conditions

Table: Important Model Equations

| | |
|----------------------------|--|
| Investment FOC | $\frac{1}{C_{it}} = \beta \mathbb{E}_t \left[\frac{1}{C_{i,t+1}} \left(\frac{r_{i,t+1}}{P_{i,t+1}} + 1 - \delta \right) \right]$ |
| Final Good equilibrium | $C_{it} + X_{it} = Y_{it}$ |
| Capital accumulation | $K_{it} = (1 - \delta)K_{i,t-1} + X_{i,t-1}$ |
| Labor market equilibrium | $L_{it}^d = \left[\frac{w_{it}}{C_{it}\psi_i} \right]^{\frac{1}{\nu}}$ |
| Capital market equilibrium | $K_{it}^d = K_{it}$ |
| Free Entry condition | $\Pi_{it} = M_{it} \frac{w_{it}}{Z_{it}} F_i^E$ |
| Current Account balance | $R_{ik,t}^X + \Pi_{ik,t}^M - R_{ki,t}^X - \Pi_{ki,t}^M = 0$ |

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Table: Calibration: USA - RoW

| Moment | Target | Model |
|------------------|--------|--------|
| Trade share | 26.4% | 26.42% |
| FDI share | 19.1% | 19.11% |
| Frac. exporters | 34.4% | 34.44% |
| Frac. affiliates | 2.7% | 2.72% |

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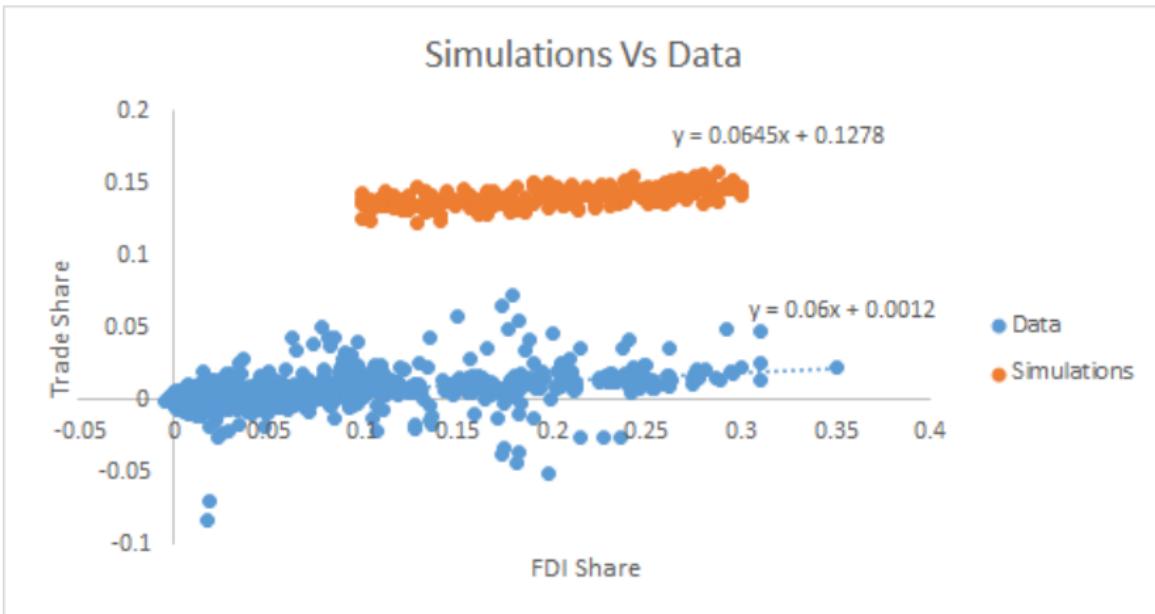
Calibration - Details

Steps in generating 200 artificial country-pairs

- 1 Generate 50-by-1 vector of FDI shares equally spaced between 10% and 30%
- 2 Stack the vector four times
 - \Rightarrow 200-by-1 vector of FDI shares
- 3 Generate 200-by-1 errors $\sim N(0, 3.2 * 10^{-5})$
- 4 Generate trade shares
 - $\text{trade} = 0.12 + 0.06 \text{ FDI} + e$
- 5 For each simulation, vary iceberg costs to match one (trade-FDI) pair

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Simulations Vs Data



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Robustness Checks

Table: Robustness - Higher Elasticity

| | (1) | (2) | (3) |
|-----------------|--------|--------|--------|
| β_{trade} | 0.28 | | 0.08 |
| | (0.02) | | (0.00) |
| β_{FDI} | | 0.06 | 0.05 |
| | | (0.00) | (0.00) |

Results from estimating trade-comovement regression with model implied numbers for 200 artificial country-pairs. σ , the elasticity of substitution between varieties, is 4.

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Robustness Checks

Table: Robustness - Lower Elasticity

| | (1) | (2) | (3) |
|-----------------|--------|--------|--------|
| β_{trade} | 0.28 | | 0.09 |
| | (0.02) | | (0.00) |
| β_{FDI} | | 0.06 | 0.05 |
| | | (0.00) | (0.00) |

Results from estimating trade-comovement regression with model implied numbers for 200 artificial country-pairs. σ , the elasticity of substitution between varieties, is 3.

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Robustness Checks

Table: Robustness - Vary intercept

| | (1) | (2) | (3) |
|-----------------|--------|--------|--------|
| β_{trade} | 0.22 | | 0.07 |
| | (0.02) | | (0.00) |
| β_{FDI} | | 0.06 | 0.05 |
| | | (0.00) | (0.00) |

Results from estimating trade-comovement regression with model implied numbers for 200 artificial country-pairs. The FDI-trade intercept is 33% lower than the baseline.

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Robustness Checks

$$Y_{ik,t}(\varphi) = \frac{Z_{i,t}^\mu Z_{k,t}^{1-\mu}}{h_{ik}} \varphi K_{k,t}^\alpha(\varphi) L_{k,t}^{1-\alpha}(\varphi)$$

Table: Robustness - Tech. Transfer

| | $\mu = 0\%$ | | | $\mu = 30\%$ | | |
|----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\log(\text{trade})$ | 0.33*** (0.05) | | 0.07 (0.05) | 1.26*** (0.11) | | 0.38*** (0.05) |
| $\log(\text{FDI})$ | | 0.08*** (0.01) | 0.07*** (0.01) | | 0.28*** (0.01) | 0.25*** (0.01) |

Results from allowing multinationals to transfer technology.

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Intuition - Mechanism

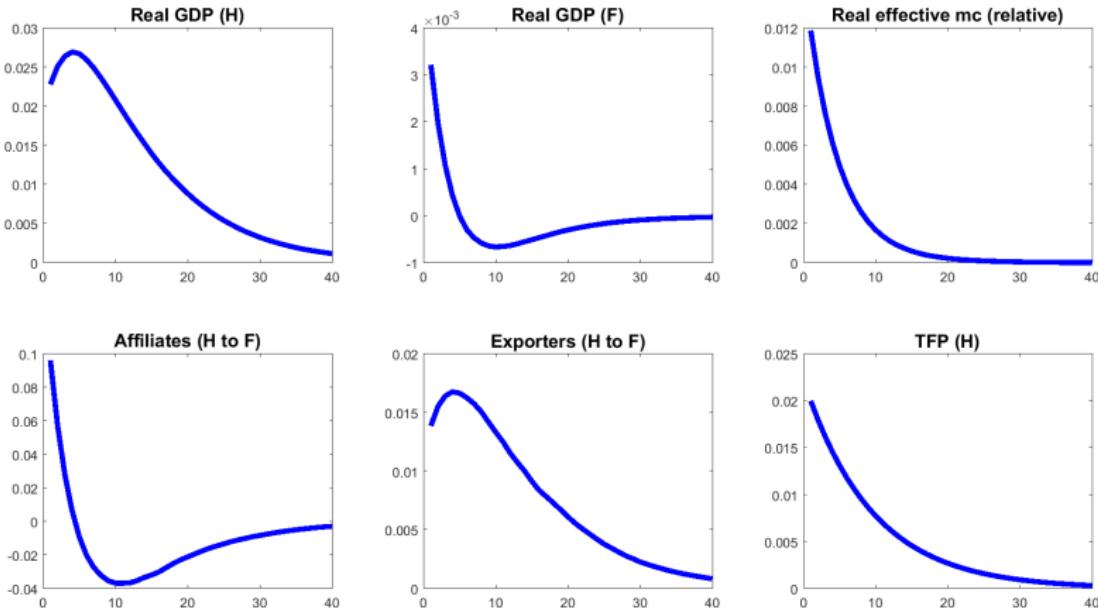


Figure: Impulse responses for a one-standard-deviation shock to Home TFP.

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Intuition - Varying μ

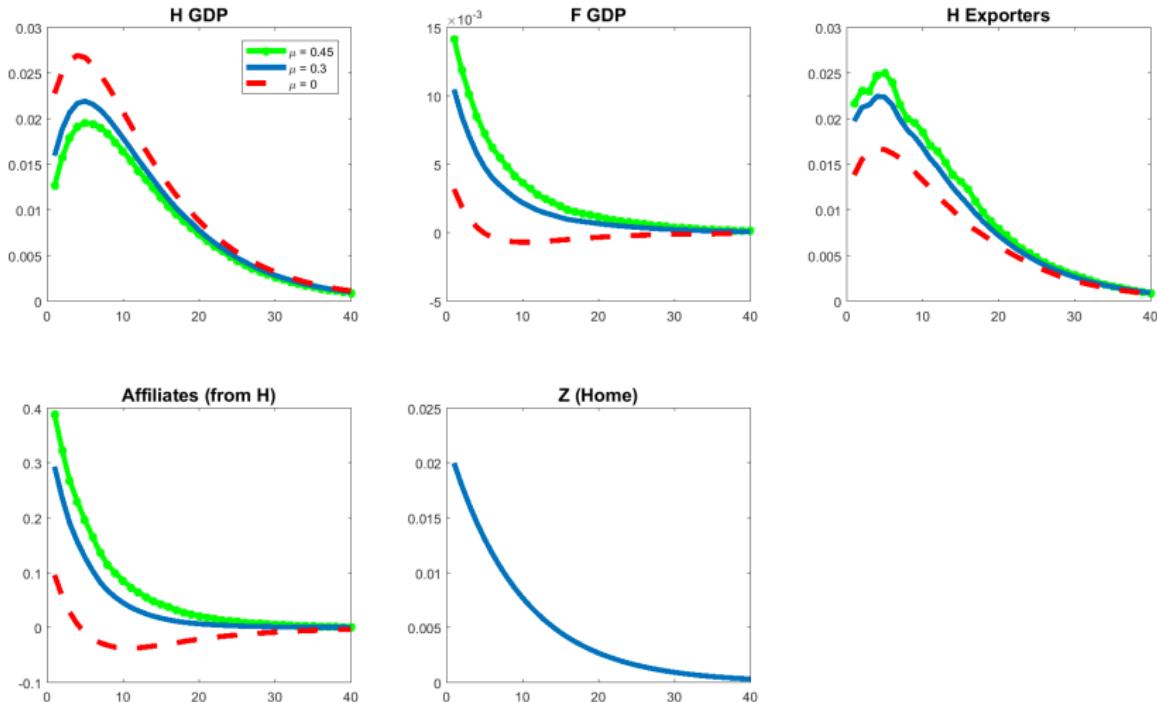


Figure: Comparison of impulse responses in a models with different μ values.

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Comparison - Models with and without FDI

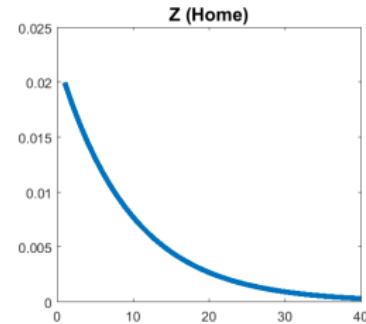
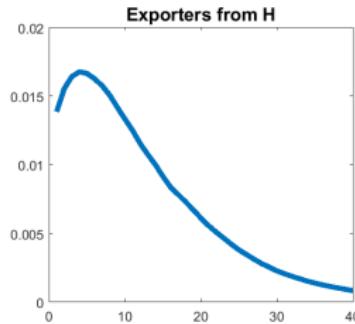
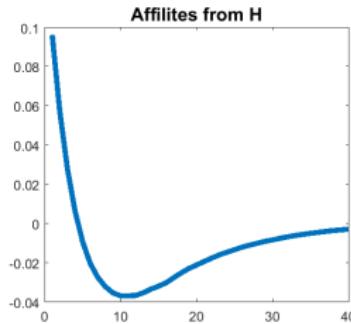
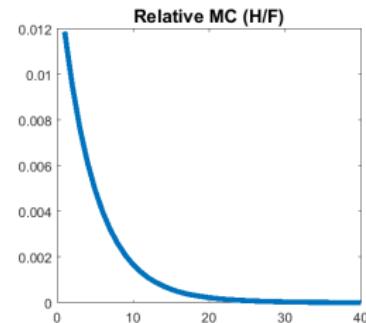
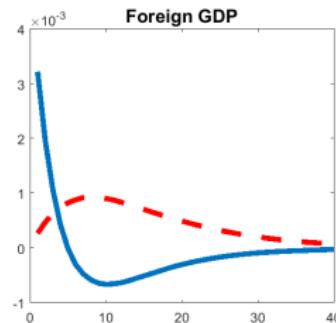
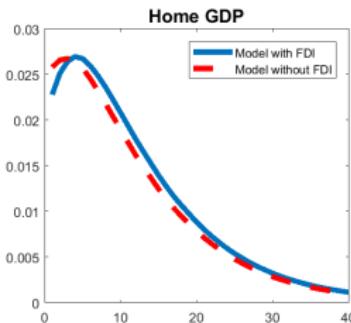


Figure: Comparison of impulse responses in a model with and without FDI.

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Vary trade iceberg cost

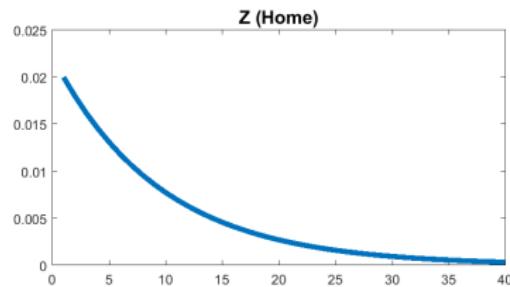
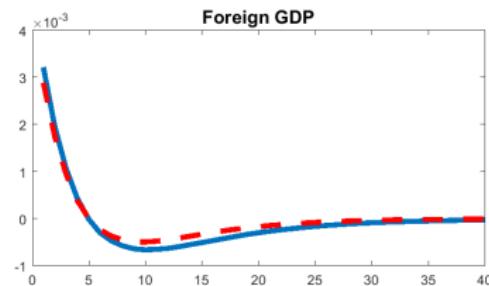
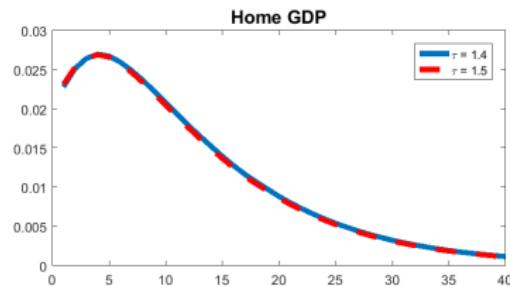


Figure: Comparison of impulse responses in a model with high and low τ .

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Vary FDI efficiency cost

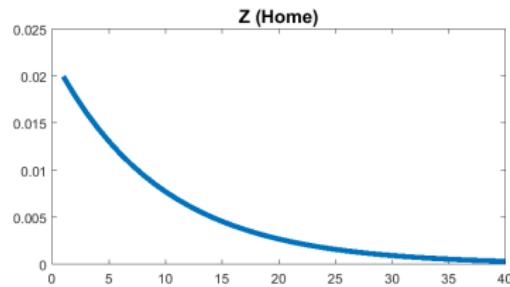
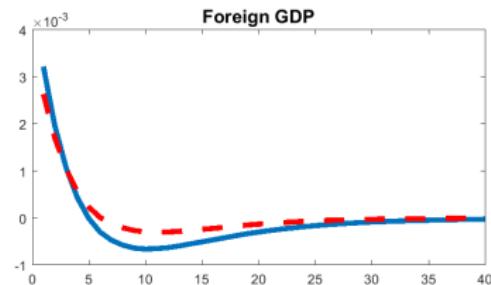
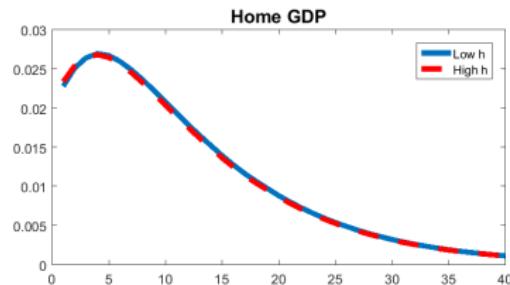


Figure: Comparison of impulse responses in a model with high and low h .

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EM in Data: Exporters and Multinats.

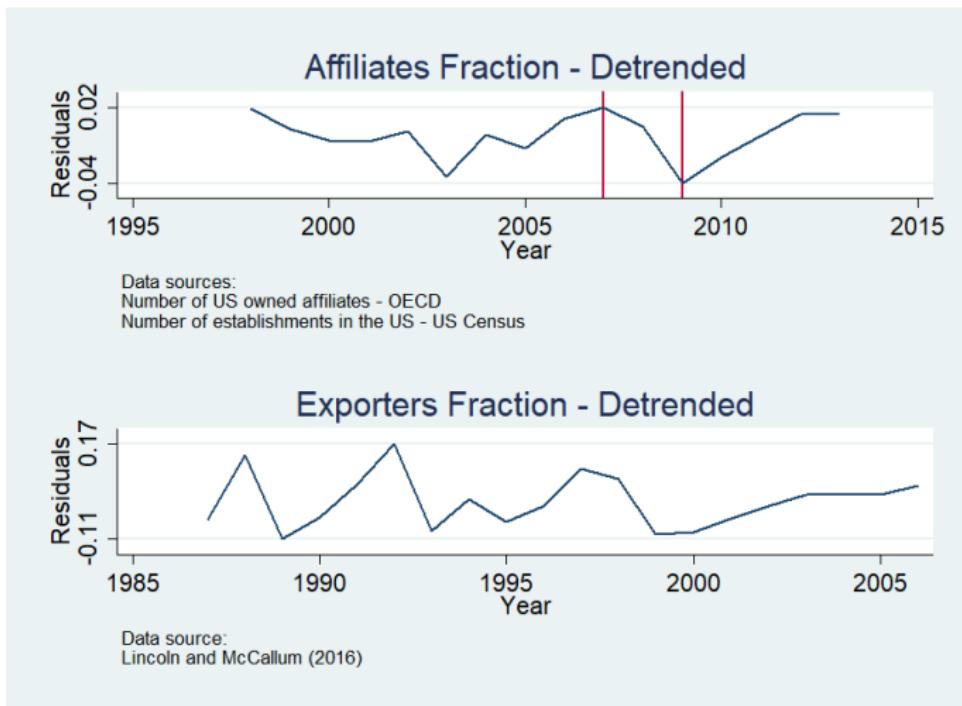


Figure: Extensive margin variation over the business cycle for exporters and affiliates.

Calibration

Simulation results

Mechanisms

EM - Model Vs. Data

Table: Volatility of Affiliate Share, Model Vs Data

| | Model | Data |
|----------------------------------|-------|---------------|
| | Mean | Max |
| Residual diff, peak to trough | 9.5% | 21.3% 6.3% |

Difference in residuals from log-linear detrended affiliates shares. In the data, differences are computed between peak and trough in the US. In the model, I average the differences in log affiliate shares between peak and trough over 100 simulations with 20 periods in each simulation.

Calibration

Simulation results

Households

- Same instantaneous utility function as before, but households can save in a risky asset.
- Households do not own capital, so can not invest in it.

$$U(s_0) = \max_{C(s^t), L(s^t), B(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t | s_0) \left[\log(C(s^t)) - \psi \frac{L^{1+\chi}}{1+\chi} \right]$$

subject to,

$$P(s^t)C(s^t) + \sum_{s^{t+1}} Q(s^{t+1} | s^t)B(s^{t+1}) = P(s^t)W(s^t)L(s^t) + B(s^t) + \Pi(s^t)$$

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Final Good Producer

- Add home-preference
- Different elasticity of substitution between home and foreign varieties

$$D(s^t) = \left[\delta \left[\int_{i=0}^1 y_h(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} + (1 - \delta) \left[\int_{\xi^{X*}(s^t) + \xi^{F*}(s^t)} y_f(i, s^t)^\theta di \right]^{\frac{\rho}{\theta}} \right]^{\frac{1}{\rho}}$$

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Demand for Varieties

$$y_h(i, s^t) = \delta^{\frac{1}{1-\rho}} \left[\frac{P_h(i, s^t)}{P_h(s^t)} \right]^{\frac{1}{\theta-1}} \left[\frac{P_h(s^t)}{P(s^t)} \right]^{\frac{1}{\rho-1}} D(s^t)$$

$$y_f(i, s^t) = (1 - \delta)^{\frac{1}{1-\rho}} \left[\frac{P_f(i, s^t)}{P_f(s^t)} \right]^{\frac{1}{\theta-1}} \left[\frac{P_f(s^t)}{P(s^t)} \right]^{\frac{1}{\rho-1}} D(s^t)$$

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Intermediate Good Producer

- Can choose to export or conduct FDI
- If FDI, additional choice: optimizing capital across two locations

Production Function

$$y_{hh}^D(i, s^t) = A_{hh}(i, s^t) K_h^D(i, s^t)^\alpha L_h^D(i, s^t)^{1-\alpha}$$

$$y_{hf}^X(i, s^t) = A_{hf}(i, s^t) K_{hf}^X(i, s^t)^\alpha L_{hf}^X(i, s^t)^{1-\alpha}$$

$$y_{hf}^F(i, s^t) = A_{hf}(i, s^t) K_{hf}^F(i, s^t)^\alpha L_{hf}^F(i, s^t)^{1-\alpha}$$

Subject to,

$$K_h^D(i, s^t) + K_{hf}^X(i, s^t) + K_{hf}^F(i, s^t) \leq K(i, s^{t-1})$$

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Intermediate Firms - Costs

Export

- Fixed cost = F^X
- Iceberg cost = $\tau \geq 1$

FDI

- Sunk cost = F_0^F
- Fixed cost = F_1^F
- Technology transfer cost = $h \geq 1$

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Capital Reallocation for FDI Firms

FDI firms re-allocate capital depending on the aggregate states of the world in the two countries.

$$\frac{K^D}{K} = \frac{1}{1 + \left(\frac{q(s^t)W^*(s^t)}{W(s^t)} \frac{H_h^*(s^t)}{H_h(s^t)} \right)^{1/\nu} h^{(\nu-1)/\nu\alpha} e^{\frac{(\nu-1)(1-\zeta)(Z(s^t) - Z^*(s^t))}{\nu\alpha}}}$$

- Under symmetric steady state, $K^D/K = 1/2$.
- If foreign is relatively less productive, $K^D/K > 1/2$.
- Important: K^D/K independent of η .

Intermediates

Model Parameterization

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| Parameter | Description | Value |
|----------------------|---|-------|
| β | Discount factor | 0.99 |
| ψ | Preference for leisure | 4.5 |
| χ | Inverse of Frisch-elasticity | 2/3 |
| δ | Home preference | 0.757 |
| ρ | Elasticity parameter, foreign varieties | 1/3 |
| θ | Inverse markup | 0.8 |
| τ | Export iceberg cost | 1.3 |
| h | International tech. transfer cost | 1.1 |
| F^X | Export fixed cost | 0.06 |
| F_0^F | FDI sunk cost | 0.06 |
| F_1^F | FDI fixed cost | 0.12 |
| α | Capital share | 0.36 |
| δ_k | Capital depreciation rate | 0.1 |
| ζ | Home TFP share | 0 |
| M_{11}, M_{22} | Own AR1 parameter | 0.9 |
| M_{12}, M_{21} | Cross AR1 parameter | 0 |
| Ω_1, Ω_2 | std. dev., innovation to TFP | 0.1 |
| σ_η | std. dev. of iid process | 0.5 |

Steady State

Table: Benchmark Parameterization - Equilibrium

| Variable | Value |
|---------------------------------------|-------|
| Fraction, exporting firms | 9.50% |
| Fraction, FDI firms | 1.66% |
| Ratio, capital to output | 2.04 |
| Ratio, profits to output | 0.27 |
| Ratio, consumption to output | 0.80 |
| Labor | 0.32 |
| Ratio, export fixed cost to real wage | 0.18 |
| Ratio, FDI fixed cost to real wage | 0.37 |
| Ratio, FDI sunk cost to real wage | 0.18 |