



Portfolio Approach of Measuring Credit Risk

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Portfolio Credit Risk: What is that?

- **Portfolio Risk** is that loss which arises due to holding two or more assets in the portfolio
- **Concentration Risk** is the additional portfolio risk resulting from increased exposure to one obligor or groups of correlated obligors
 - Business group
 - Industry
 - Sector
 - Geographic area
- Correlation (credit quality correlation or correlation of default) is an **important driver** of portfolio credit risk.
- Credit events are not independent - firms prosper or suffer in line with each other
- When 2 or more borrowers default simultaneously, the losses are more **severe**
- The higher the correlation of default, the greater is the concentration risk of the portfolio
- The lower the correlation of default, more diversified the portfolio
- **Correlation of Default** adds to credit risk when a portfolio of loans and advances is in consideration vis-à-vis single loans or advances.
- When correlations are significant, they produce loss distributions that are highly skewed (tail measures of credit risk like value at risk captures this).



The Critical Issue in Credit Portfolio Management

- The critical element in successfully managing a credit risk portfolio is that you must manage the **dynamics of credit risk**
- **Correlated credit deterioration** has been the specific cause of many occurrences of financial distress (agricultural loans in US mid-west, oil loans in Texas, the Latin American debt crisis, recent US mortgage crisis).
- Internationally, there are lot of incidents of **clustered defaults** (within industries as well as between industries). The reason being that in addition to very significant concentrations of lending in a particular industry (e.g. energy), the regional **dependence** and a strong **correlation** between the health of the industries.
- A portfolio approach to credit risk analysis allows portfolio managers to **quantify** and stress test concentration risk along various dimensions as under.
- Managing outliers is important because a relatively small segment that disproportionately drive risk – most of the portfolio requires very little attention-**Tail Risk!!**

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Concentration of sub-prime in US

- More than half of the US's foreclosures during 2007-08 took place in 35 counties, a sign that the financial crisis devastating the national economy may have begun with collapsing home loans in only a few corners of the country depicts ripple effect of **geographic concentration**.
- Most of the 35 counties leading the foreclosure boom are clustered in places such as Southern California, Las Vegas, Phoenix, South Florida and Washington, where home values shot up dramatically in the first half of the decade, then began to crumble.
- A geographic mapping of sub-prime & Alt-A loans in the Phoenix metropolitan area has revealed that sub-prime loans are heavily concentrated in lower to middle income group of population (or small business people who don't relish disclosing all their income), newly constructed houses (many of them are refinancing) especially in urban areas (notes Anthony Saunders, 2008).
 - Source: Gwinner, William B. and Saunders, Anthony, "The Sub Prime Crisis: Implications for Emerging Markets", September 2008, The World Bank, Policy Research Working Paper 4726

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Need for Assessment of Portfolio Risk

- These examples illustrate the importance of measuring **concentration risk** in credit portfolios of banks that arises not only from exposures to a single credit, or asset class, but also from **linkages between asset classes**.
- Correlations often only become apparent when **economic conditions** turn sour! (interest rates rise and property values soften-US Mortgage Crisis).
- Banks with heavy correlation risks can make money and steer clear of trouble, so long as they hold enough **risk capital** to protect the bank against the higher level of **unexpected losses** and **charge** their customers accordingly (or exit markets where market pricing makes charging for concentration risks impossible).
- Where these skills are lacking, banks with strongly correlated portfolios suffer the **double blow** of heavy losses followed by the need to adjust ongoing business strategy

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Portfolio Risk

- In order to take advantage of credit portfolio management opportunities, the answer to these technical questions is important
 - What is the risk of a given portfolio as opposed to individual credits comprising the portfolio?
 - How do different **macroeconomic scenarios** at both the regional and the industry sector level, affect the portfolio's risk profile?
 - What is the effect of **changing the portfolio mix**?



Portfolio Mix

| | Portfolio credit quality | | | | | | | |
|--------|--------------------------|---------|------------------|---------|------------------|---------|------------------|---------|
| | High | | Average | | Low | | Very Low | |
| Grades | No. of Borrowers | % Share | No. of Borrowers | % Share | No. of Borrowers | % Share | No. of Borrowers | % Share |
| AAA | 191 | 3.82% | 146 | 2.92% | 50 | 1.00% | 25 | 0.50% |
| AA | 295 | 5.90% | 250 | 5.00% | 77 | 1.54% | 51 | 1.02% |
| A | 1463 | 29.26% | 669 | 13.38% | 185 | 3.70% | 158 | 3.16% |
| BBB | 1896 | 37.92% | 1558 | 31.16% | 827 | 16.54% | 660 | 13.20% |
| BB | 954 | 19.08% | 1622 | 32.44% | 1903 | 38.06% | 1780 | 35.60% |
| B | 136 | 2.72% | 556 | 11.12% | 1618 | 32.36% | 1851 | 37.02% |
| CCC | 65 | 1.30% | 199 | 3.98% | 340 | 6.80% | 475 | 9.50% |
| Total | 5000 | 100.00% | 5000 | 100.00% | 5000 | 100.00% | 5000 | 100.00% |

Source: M B Gordy, Journal of Banking & Finance, Vol. 24 (2000), pp. 119-149



What are Credit Risk Losses?

- If credit can be defined as “nothing but the expectation of a sum of money within some limited time”, then credit risk is “the chance that expectation will not be met”.
- Expected Credit Risk Loss (EL)** is intended to set reserve requirements for doubtful accounts, calculation of PLR (default premium), pricing credit risky instruments (bonds and exotic options), and for calculation of risk adjusted profitability (e.g. **RAROC**).

$$EL = EAD \times PD \times LGD$$

- The bank can also suffer losses in excess of expected losses, say, during economic downturns. These losses are called **Unexpected Losses (UL)** or uncertain losses.

$$UL = EAD \times \sqrt{PD \times \sigma_{LGD}^2 + LGD^2 \times \sigma_{PD}^2}$$

- The **capital** base is required to absorb the **unexpected losses (UL)**, as and when they arise.



Expected Vs. Unexpected Loss

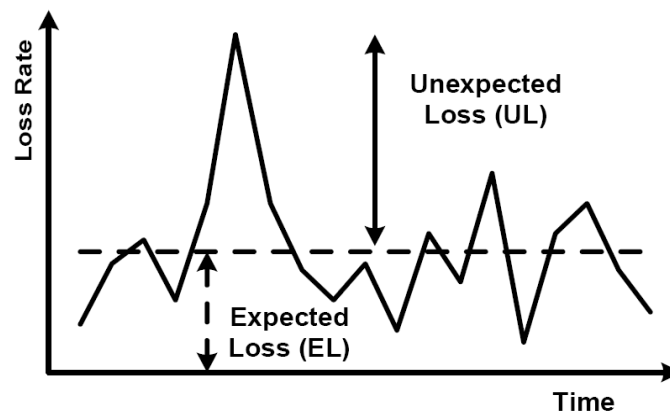
- While it is never possible to know in advance the losses a bank will suffer in a particular year, a bank can forecast the average level of credit losses (EL) it can reasonably expect to experience.
- Losses above the expected levels are usually referred to as unexpected losses (UL).
- Institutions know that these losses will occur now and then, but they cannot know in advance the time of their arrival and their severity.
- Banks are in general expected to cover their EL on an ongoing basis, e.g. by pricing, provisions and write-offs, because it represents just another cost component of the lending business.
- According to this concept, capital is only needed for covering unexpected losses. Hence, in Basel II, the banks are only required to hold capital against UL.
- **Capital is the cushion** that protects the liability holders of a bank.

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EL Vs. UL

Figure 1



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Aims of the Credit Portfolio Management

- Credit portfolio management aims to increase efficiency and return by:
 1. identifying credit concentrations
 2. calculating the exposure arising from those risks
 3. assisting management in allocating economic capital to internal businesses (**capital budgeting**)
 4. identifying the **marginal contribution** that a particular credit adds to risk and what return compensates the institution for it; and
 5. deciding which assets to hedge, sell or securities.

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Loan Evaluation and Monitoring

- Existence of a sound Loan Monitoring system is an anathema in Indian Banking
 - The **Transition Matrix** provides the profile of credit quality changes or migrations that have taken place for the selected credit portfolio between any two years that are selected
 - The transition matrix including probabilities to move from one rating to another rating represents the kernel of many credit risk and rating calculations
 - Transition matrix is a promising tool for credit portfolio capital allocation, loan monitoring and performance evaluation



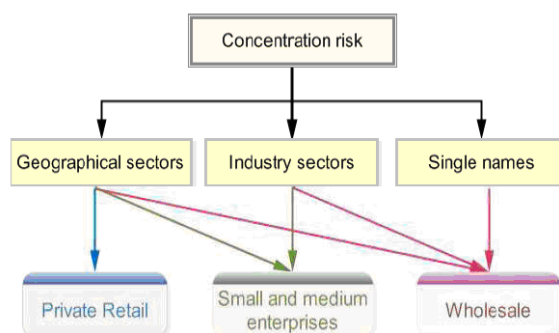
Rating Transition of 572 Corporate Bonds Rated Externally by CRISIL (Benchmarking)

One Year Average Rating Transition Matrix for the Period 1992-2009 in %

| | | Year T+1 | | | | | | | |
|--------|-----|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--------|
| | | AAA | AA | A | BBB | BB | B | CCC | D |
| Year T | AAA | 96.05% | 3.95% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| | AA | 2.81% | 89.74% | 6.20% | 0.68% | 0.39% | 0.10% | 0.00% | 0.10% |
| | A | 0.00% | 3.99% | 83.71% | 7.12% | 2.70% | 0.32% | 0.54% | 1.62% |
| | BBB | 0.00% | 0.51% | 5.09% | 75.83% | 10.69% | 1.53% | 2.80% | 3.56% |
| | BB | 0.00% | 0.70% | 0.00% | 1.41% | 59.86% | 3.52% | 7.75% | 26.76% |
| | B | 0.00% | 0.00% | 0.00% | 7.41% | 0.00% | 40.74% | 22.22% | 29.63% |
| | CCC | 0.00% | 0.00% | 0.00% | 2.13% | 0.00% | 0.00% | 53.19% | 44.68% |



Portfolio Dimension of Credit Risk





Industry Concentration of a Bank's Loan Portfolio

Table 11: Summary Table for Industry wise March 2010 Loan Outstanding Positions (of A/c's >5 Cr. exposure)

| SL# | Industry Cat | N | mean | p50 | p75 | p99 | max | cv | sk | kurt | Gini | Theil | Rating | Region |
|-------|----------------------------|------|---------|--------|--------|----------|----------|-------|-------|---------|-------|-------|------------|-----------------|
| 1 | All Engineering | 94 | 9.445 | 4.625 | 10.500 | 77.500 | 77.500 | 1.460 | 2.816 | 11.637 | 0.579 | 0.613 | | |
| 2 | Auto & Parts | 58 | 13.552 | 7.242 | 15.154 | 120.227 | 120.227 | 1.376 | 3.843 | 20.781 | 0.538 | 0.554 | A | MUM(S) |
| 3 | Cement & Pdcts & Ceramics | 29 | 33.368 | 7.286 | 21.988 | 348.172 | 348.172 | 2.200 | 3.330 | 13.694 | 0.736 | 1.128 | C,A+ | HYD,ND |
| 4 | Chemical | 66 | 25.923 | 6.176 | 11.119 | 399.964 | 399.964 | 2.368 | 4.141 | 23.005 | 0.757 | 1.238 | UR,B | MUM(S) |
| 5 | Construction & Real Estate | 261 | 26.629 | 6.071 | 16.079 | 459.258 | 1000.000 | 3.098 | 7.935 | 82.340 | 0.762 | 1.364 | A+,A+,A | GHAZ,DEL,MUM(N) |
| 6 | Educational Serv | 105 | 10.126 | 5.692 | 10.695 | 66.685 | 72.772 | 1.279 | 2.947 | 12.512 | 0.513 | 0.493 | | |
| 7 | Ferrous Metals | 230 | 20.648 | 7.539 | 18.101 | 198.006 | 497.384 | 2.230 | 6.583 | 59.227 | 0.660 | 0.926 | B+,B+,A+ | KOL,GHAZ,MUM(S) |
| 8 | Film & Entertainment | 7 | 36.536 | 10.683 | 73.730 | 85.727 | 85.727 | 1.020 | 0.335 | 1.212 | 0.496 | 0.486 | | |
| 9 | Financial Services | 31 | 64.907 | 0.224 | 42.909 | 1000.351 | 1000.351 | 2.869 | 4.351 | 22.047 | 0.749 | 1.160 | UR,A,B+ | MUM(S) |
| 10 | Food & Pdcts | 143 | 16.197 | 7.540 | 16.827 | 174.992 | 183.432 | 1.579 | 4.183 | 24.800 | 0.581 | 0.654 | B+,B+ | ND |
| 11 | Gems & Jewellery | 36 | 9.731 | 6.470 | 12.261 | 49.164 | 49.164 | 1.183 | 2.009 | 7.010 | 0.455 | 0.357 | | |
| 12 | Infrastructure | 181 | 93.055 | 36.395 | 98.749 | 1012.789 | 1164.040 | 1.763 | 3.610 | 19.009 | 0.426 | 0.305 | B+,B+,A,A+ | PAT,CHAND,JAIP |
| 13 | IT Software & BPO | 16 | 10.202 | 6.568 | 13.411 | 32.247 | 32.247 | 0.999 | 1.186 | 3.104 | 0.677 | 0.878 | | |
| 14 | Leather & Leather Pdcts | 7 | 5.165 | 5.827 | 7.680 | 14.868 | 14.868 | 1.022 | 0.752 | 2.627 | 0.312 | 0.177 | | |
| 15 | Misc. Industries | 197 | 25.163 | 4.850 | 9.950 | 961.284 | 1543.770 | 5.216 | 9.817 | 105.242 | 0.824 | 2.066 | B,B+,C | MUM(S),ND |
| 16 | NBFCs | 34 | 118.544 | 36.927 | 97.135 | 1699.997 | 1699.997 | 2.618 | 4.344 | 21.717 | 0.736 | 1.238 | A,A++,A+ | MUM(S),DEL |
| 17 | Other Metal | 27 | 18.716 | 6.793 | 13.889 | 273.787 | 273.787 | 2.749 | 4.762 | 24.134 | 0.684 | 1.243 | UR | MUM(S) |
| 18 | Paper & Pdcts | 54 | 17.083 | 8.857 | 16.000 | 164.014 | 164.014 | 1.541 | 3.686 | 19.526 | 0.589 | 0.656 | B+ | AMR IT |
| 19 | Plastic & Pdcts | 38 | 8.063 | 5.562 | 9.416 | 36.140 | 36.140 | 0.992 | 1.725 | 5.720 | 0.469 | 0.377 | | |
| 20 | Services | 72 | 17.442 | 10.622 | 25.139 | 71.165 | 71.165 | 1.007 | 1.289 | 3.791 | 0.514 | 0.439 | | |
| 21 | Sugar | 18 | 63.884 | 25.683 | 53.263 | 500.040 | 500.040 | 1.849 | 3.058 | 11.664 | 0.675 | 0.917 | A,C | GHAZ,ND |
| 22 | Textiles | 170 | 21.639 | 7.924 | 24.230 | 154.080 | 215.999 | 1.525 | 3.027 | 13.714 | 0.623 | 0.719 | C | ND |
| 23 | Timber Plywood | 28 | 2.780 | 1.374 | 3.825 | 21.440 | 21.440 | 1.460 | 3.635 | 17.249 | 0.543 | 0.573 | | |
| Total | | 1902 | 29.430 | 7.015 | 20.336 | 449.995 | 1699.997 | 3.223 | 9.667 | 126.498 | 0.751 | 1.345 | | |



Expected Loss (EL) based HHI Measure of Portfolio Concentration

Besides Rating, EL based HHI Measure depends upon pool size, Largest loan size as well as the no. of loans.

Hence, Portfolio slicing is important!

Figure 3

HHI Measures Increase in Concentration

| Pool | Loans (#) | Pool Size (Millions) | Average Loan Size | Largest Loan Size | HHI |
|------|-----------|----------------------|-------------------|-------------------|-----|
| 1 | 500 | \$275 | \$550,000 | \$550,000 | 20 |
| 2 | 100 | \$55 | \$550,000 | \$550,000 | 100 |
| 3 | 100 | \$58 | \$584,500 | \$4,000,000 | 352 |

$$HHI = \text{Sum} (((\$ \text{ Loan Expected Loss} / \$ \text{ Pool Expected Loss}) * 100)^2).$$

- In fraction, HHI of Pool 1 is 0.020 means well diversified; Pool 2: 0.10 moderate level of concentration and Pool 3: 0.352 indicate high level of concentration
- However, HHI does not tell the ways to reduce concentration!



Impact of Correlation of Default: The Third Dimension of Credit Risk

- Expected Loss of a Portfolio is the algebraic sum of the Expected Losses of the individual credits (linear)

$$EL_p = \sum_{i=1}^n EL_i$$

- However, Unexpected Loss of a Portfolio is not the algebraic sum of the Unexpected Losses of the individual credits!

$$UL_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{i,j}^d \times UL_i \times UL_j}$$

Because of diversification effects due to correlation, observe that

$$UL_p << \sum_i UL_i$$

$$UL_p = \sum_i RC_i$$

- Correlation of default $\rho_{i,j}^d$ is very important when assessing the true risk of a portfolio as quantified by the portfolio's Unexpected Loss UL_p



Risk Contribution

$$\rho_{ab} < 1, \\ \sigma_{a+b} < \sigma_a + \sigma_b$$

$$\{\sigma_a = UL_a \ \& \ \sigma_b = UL_b\}$$

- ♦ Only a portion of individual assets unexpected loss contributes to the unexpected loss of the portfolio → RISK CONTRIBUTION

- ♦ Risk Contribution is the incremental risk contributed by an individual asset to the portfolio (also termed as MRC)

- ♦ It is a measure of the contribution of default risk in the portfolio

$$\text{♦ } MRC_i = \sqrt{\rho} \times UL_i$$

- ♦ The Portfolio unexpected loss is the sum of the risk contributions of individual assets



UL and Marginal Risk Contribution

- Risk contribution measures the portion of individual asset's (or industry/sector/region) UL_i contributes to the portfolio risk (UL_p).
- Risk contribution (MRC or URC) is defined by:

$$MRC_i \equiv UL_i \frac{\Delta UL_p}{\Delta UL_i}$$

$$= \frac{UL_i \sum_j UL_j \rho_{ij}}{UL_p}$$

- The above expression measures the unexpected loss contribution of a single asset to its portfolio (i.e. Change in portfolio risk due to one unit change risk of an individual asset).

Also note that: $UL_p = \sum_i MRC_i$

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Measuring Default Correlation (DC)

Difficulties

- Lack of sufficient **historical default data** and data on **credit quality co-movements** makes it difficult to measure directly
- Pair-wise default correlation for even a medium sized portfolio will require a large number of calculations

Solution

- Identify the basic causes of default correlation
- Single Default Correlation can be estimated by tracking the Banks' historical **NPA movements**.
- Approximate default correlations using a **proxy approach** (equity correlation-Company Correlation, Industry Correlation)
- From CRAs **published rating history** DC can be estimated



Estimation of Single Default Correlation

- In estimating the loss correlation from historical NPA data of the Bank, we can first assume that all the loans were identical in terms of risk characteristics to create a **single pool**.
- We know that portfolio Unexpected Loss of a large number of loan pool is represented by:

$$UL_P = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{i,j}^d \times UL_i \times UL_j}$$

- We can get an estimate of average correlation if we assume correlation between each loan is identical (i.e. $\rho_{ij} = \rho$)

$$\rho \sum_{i=1}^N \sum_{j=1}^N UL_i UL_j$$

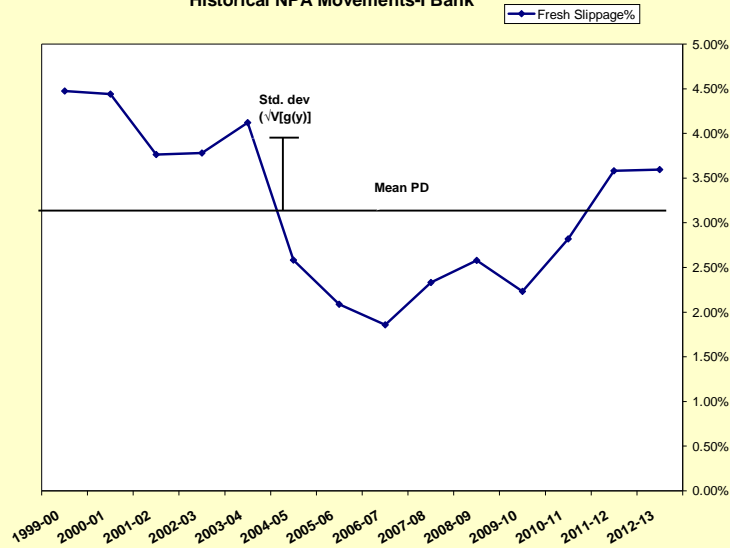
- Assuming each loan has same UL (i.e. $UL_i = UL_j$) we can estimate the correlation using the following expression

$$\rho = \frac{UL_P^2}{(\sum_{i=1}^N UL_i)^2} = \frac{UL_P^2}{(N^2 \times UL_i^2)} = \frac{UL\%_P^2}{UL\%_i^2}$$

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Historical NPA Movements-I Bank





Single Default Correlation Estimates for Banks

| Bank Name | PD% | UL% | DC | AC | totalbuss in Rs. Crore | Branches |
|---------------------------|-------|-------|-------|--------|------------------------------|----------|
| Allahabad Bank | 2.80% | 1.46% | 0.79% | 4.76% | 225512 | 2415 |
| Andhra Bank | 1.60% | 0.94% | 0.73% | 4.92% | 163591 | 1657 |
| Axis Bank Ltd. | 1.80% | 1.11% | 0.70% | 5.56% | 331646 | 1390 |
| Bank of Baroda | 2.01% | 1.27% | 0.81% | 5.95% | 534115 | 3490 |
| Bank of India | 2.34% | 0.98% | 0.42% | 2.94% | 511982 | 3728 |
| Canara Bank | 2.59% | 0.95% | 0.36% | 2.38% | 506440 | 3212 |
| Central Bank of India | 2.76% | 1.57% | 0.92% | 5.55% | 309081 | 3728 |
| Corporation Bank | 1.35% | 0.62% | 0.28% | 2.92% | 203597 | 1300 |
| Dena Bank | 4.21% | 2.84% | 2.01% | 8.82% | 109038 | 1291 |
| Federal Bank | 3.19% | 1.57% | 0.80% | 4.44% | 74968 | 743 |
| HDFC Bank Ltd. | 2.05% | 1.10% | 0.60% | 4.47% | 368569 | 1986 |
| ICICI Bank Ltd. | 3.25% | 2.92% | 2.71% | 13.02% | 441968 | 2533 |
| IDBI Bank Ltd. | 1.05% | 1.28% | 1.58% | 14.54% | 337584 | 816 |
| ING Vysya Bank Ltd. | 2.49% | 0.69% | 0.19% | 1.35% | 53796 | 510 |
| Indian Bank | 0.73% | 0.65% | 0.58% | 8.16% | 181054 | 1932 |
| Indian Overseas Bank | 2.92% | 1.02% | 0.36% | 2.23% | 257062 | 2600 |
| IndusInd Bank Ltd. | 3.41% | 2.63% | 2.10% | 10.26% | 60531 | 300 |
| Jammu & Kashmir Bank Ltd. | 1.56% | 0.39% | 0.10% | 1.00% | 70870 | 591 |
| Karnataka Bank Ltd. | 2.63% | 1.72% | 1.16% | 7.00% | 44684 | 479 |
| Karur Vysya Bank Ltd. | 1.76% | 1.39% | 1.12% | 8.50% | 42536 | 369 |
| Kotak Mahindra Bank Ltd. | 1.81% | 1.34% | 1.01% | 7.66% | 58590 | 321 |
| Oriental Bank of Commerce | 3.22% | 2.58% | 2.13% | 10.69% | 234962 | 1620 |
| Punjab National Bank | 2.58% | 1.15% | 0.52% | 3.43% | 555006 | 5100 |
| State Bank of India | 2.96% | 0.90% | 0.28% | 1.71% | 1690652 | 13542 |
| UCO Bank | 2.69% | 0.89% | 0.31% | 2.00% | 244349 | 2202 |
| United Bank of India | 1.97% | 0.80% | 0.33% | 2.64% | 131347 | 1597 |
| Vijaya Bank | 2.86% | 0.74% | 0.19% | 1.24% | 121967 | 1200 |

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Estimation of Single Default Correlation...

- In real life portfolio, this is not the case and we have an idea of the distribution of the creditworthiness of the loans in the portfolio.
- Accordingly, one should use region wise/industry wise loan distribution and estimate sum of the exposure weighted ULs of the individual loans according to their allocation to each region/industry group:

$$\sum_{i=1}^N UL_i = w_1 \sum_{i=1}^{G1} UL_i + w_2 \sum_{i=G1+1}^{G2} UL_i + w_3 \sum_{i=G2+1}^{G3} UL_i + \dots + w_n \sum_{i=Gn+1}^N UL_i$$

- See the Excel Illustration

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Estimation of Concentration Risk capital for Exposure to top 20 borrower

Marginal risk contribution (MRC_i)

$$= \sqrt{r_i} \times UL_i \times \text{Outstanding}$$

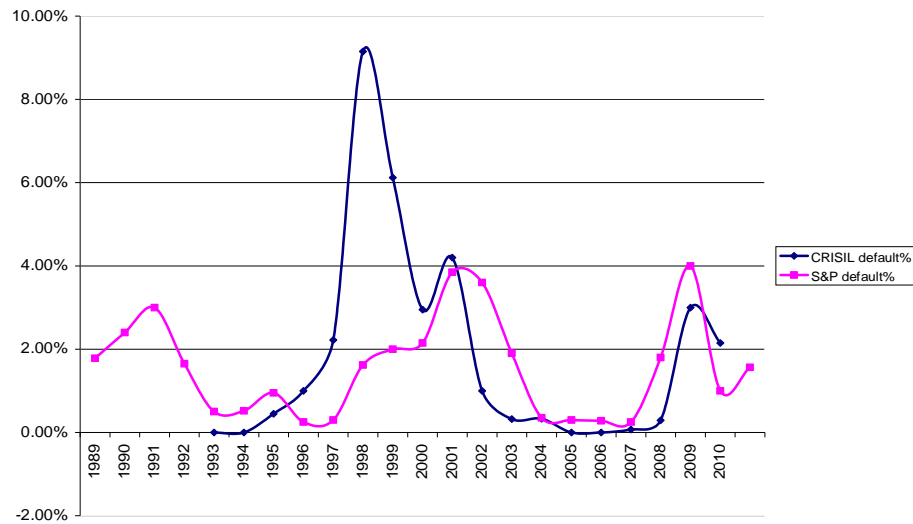
| Borrower Code | Total Sanctioned | Limit % to capital funds | Total Credit Exposure | Borrower Rating | % to total credit exposure | UL% | Default Correlation | MRC in Rs. Cr. |
|---------------|------------------|--------------------------|-----------------------|-----------------|----------------------------|--------|---------------------|----------------|
| 1 | 125000 | 12.16% | 1900 | CR-3 | 1.99% | 5.19% | 0.003 | 5.34 |
| 2 | 163500 | 15.91% | 1635 | CR-2 | 1.71% | 3.22% | 0.004 | 3.41 |
| 3 | 120759 | 11.75% | 1309.65 | CR-4 | 1.37% | 6.38% | 0.006 | 6.63 |
| 4 | 120672 | 11.74% | 1270 | CR-2 | 1.33% | 3.22% | 0.004 | 2.65 |
| 5 | 127000 | 12.36% | 1268.86 | CR-4 | 1.33% | 6.38% | 0.006 | 6.42 |
| 6 | 88000 | 8.56% | 1250 | CR-3 | 1.31% | 5.19% | 0.003 | 3.51 |
| 7 | 100087 | 9.74% | 1244.36 | CR-3 | 1.30% | 5.19% | 0.003 | 3.50 |
| 8 | 122500 | 11.92% | 1230.71 | CR-3 | 1.29% | 5.19% | 0.003 | 3.46 |
| 9 | 124436 | 12.11% | 1225 | CR-3 | 1.28% | 5.19% | 0.003 | 3.44 |
| 10 | 190000 | 18.49% | 1207.59 | CR-3 | 1.26% | 5.19% | 0.003 | 3.39 |
| 11 | 130965 | 12.74% | 1206.72 | CR-2 | 1.26% | 3.22% | 0.004 | 2.52 |
| 12 | 77720 | 7.56% | 1200 | CR-5 | 1.26% | 7.61% | 0.005 | 6.34 |
| 13 | 123071 | 11.98% | 1105 | CR-5 | 1.16% | 7.61% | 0.005 | 5.93 |
| 14 | 103749 | 10.10% | 1037.49 | CR-4 | 1.09% | 6.38% | 0.006 | 5.25 |
| 15 | 126886 | 12.35% | 1000.87 | CR-2 | 1.05% | 3.22% | 0.004 | 2.09 |
| 16 | 74200 | 7.22% | 900.99 | CR-5 | 0.94% | 7.61% | 0.005 | 4.76 |
| 17 | 120000 | 11.68% | 880 | CR-4 | 0.92% | 6.38% | 0.006 | 4.46 |
| 18 | 110500 | 10.75% | 788.27 | CR-1 | 0.82% | 4.02% | 0.011 | 3.30 |
| 19 | 78827 | 7.67% | 777.2 | CR-6 | 0.81% | 12.25% | 0.004 | 5.75 |
| 20 | 90099 | 8.77% | 742 | CR-6 | 0.78% | 12.25% | 0.004 | 5.49 |
| 21 | 2317972 | | 23179.71 | | 24.26% | | | 87.55 |

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Default Rates and Business Cycle

Overall Annual Default Rates



Source: CRISIL



Estimation of Correlation through Equity Data

- Equity Correlation Can be used as proxy for Asset Correlation & Default Correlation
- In this case, equity correlation of industry index return ($\rho_{\alpha\beta}$) has been taken as proxy for asset correlation.
- The correlation of asset returns between two industries (A & B) is:

$$\rho(A, B) = \omega_1^A \times \omega_1^B \times \rho_{\alpha\beta}$$

- Weights are the obligor specific risk (can be represented by their market size (%) or asset size (%)).
- We have used regression equation on equity return of industry stock indexes over the market index (BSE 500) return and find obligor specific risk weight (or idiosyncratic) by using: $1-R^2$; where R^2 (captures systematic risk) is the ESS/TSS. For this, we have used 10 years of monthly equity returns data from February 1999 till March 2009.



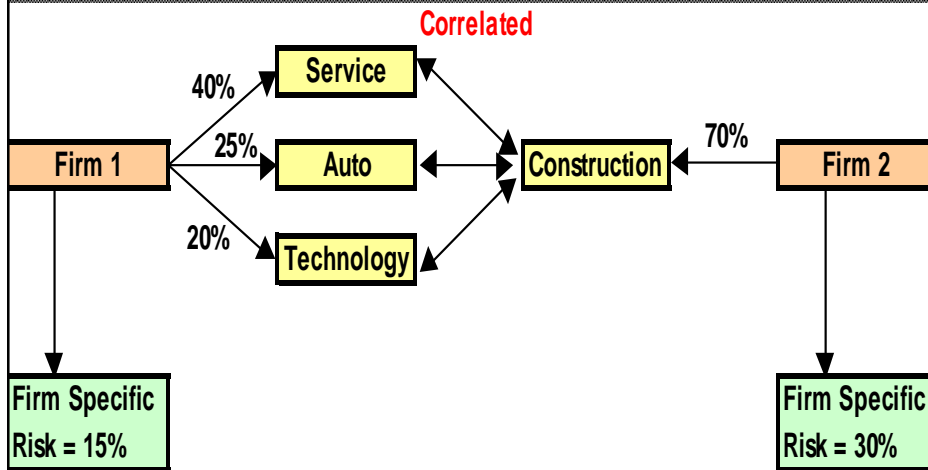
Calculation of Implied Asset Correlation

- Estimate **systematic risk** of each loan – the relationship between equity returns and returns on market/industry indices.
- Estimate the **equity correlation** between each pair of market/industry indices.
- Calculate the **implied asset correlation** coefficient as the weighted average of the systematic risk factors x the index correlations.



Correlation Effect on Portfolio Risk

Correlation



Calculation of Asset & Default Correlation from equity returns

- Vivendi Universal is a co. involved in mobile telecom (T) and leisure (L). General Motors in the world's largest car manufacturer (industry C).
- The analyst considers that exposure of Vivendi is the following based on equity return OLS:
 - $A_{viv} = 0.4T + 0.2L + \text{error}_{viv}$
 - while that for General Motors is:
 - $A_{GM} = 0.9C + \text{error}_{GM}$
- Both are obtained from equity returns (Co. vs. industry)



Correlation Estimation: Ex..

| Sectors | T | L | C |
|---------|------|------|------|
| T | 100% | 30% | 10% |
| L | 30% | 100% | 10% |
| C | 10% | 10% | 100% |

One can immediately calculate the two firms' asset correlation:

$$\text{Corr}(A_{\text{viv}}, A_{\text{GM}}) = \text{Cov}(A_{\text{viv}}, A_{\text{GM}}); \quad \{\text{Assuming asset return have unit variance}\}$$

$$\begin{aligned} &= 0.4 \times 0.9 \times \text{Cov}(T, C) + 0.2 \times 0.9 \times \text{Cov}(L, C) \\ &= 0.4 \times 0.9 \times 10\% + 0.2 \times 0.9 \times 10\% \\ &= 5.4\% \end{aligned}$$



Estimate Correlation..Exercise

- Estimate the systematic risk of each company by regressing the stock returns for each company on the relevant market/industry indices.
- $R_A = .9R_{\text{CHEM}} + U_A$
- $R_Z = .74R_{\text{INS}} + .15R_{\text{BANK}} + U_Z$
- $\rho_{A,Z} = (.9)(.74)\rho_{\text{CHEM},\text{INS}} + (.9)(.15)\rho_{\text{CHEM},\text{BANK}}$
- Estimate the correlation between the indices.
- If $\rho_{\text{CHEM},\text{INS}} = .16$ and $\rho_{\text{CHEM},\text{BANK}} = .08$, then $\rho_{AZ} = ?$



Obtaining JDP from Asset Correlation through Copula

- At the end of 2002, Vivendi and GM were rated BB and BBB, respectively by S&P.
- The 1-year average default rates for issuers in BB and BBB categories (mapped from Transition Matrix), we find 1.53% and 0.39%.
- $JDP = N_2(pd_{viv}, pd_{GIM}, 5.4\%)$
- $= N_2[N^{-1}(1.53\%), N^{-1}(0.39\%, 5.4\%)]$
- $= \text{bivnor}(-2.16223587, -2.660606739, 5.4\%)$
- $= 0.00008777$
- Finally, the **default correlation** is obtained $= 0.37\%$ (using the formula-1)
- Note that JDP is an important determinant of DC

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Industry wide Loan Portfolio Distribution: Comparison of System & Two Systematically Important Banks in India

| | Industry | Western Side Large Bank | | South Side Medium Bank | | System as a whole | | |
|----|-------------------------------|----------------------------|--------|---------------------------|--------|-------------------|----------------|---------------------------------|
| | | Exposure Share% | GNPA% | Exposure Share% | GNPA% | Industry PD% | Industry DD | Idiosyncratic Risk (1-R2) |
| 1 | Automobile | 2.78% | 0.26% | 2.57% | 1.86% | 2.70% | 3.60 | 34.45% |
| 2 | Cement | 1.45% | 0.37% | 3.63% | 1.20% | 1.22% | 3.77 | 52.06% |
| 3 | Chemical | 2.40% | 15.26% | 5.39% | 2.17% | 3.47% | 4.42 | 14.69% |
| 4 | Coal & Mining & Lignite | 0.12% | 2.77% | 0.20% | 9.05% | 3.73% | 1.53 | 54.90% |
| 5 | Computer Software | 0.03% | 33.08% | 0.32% | 1.31% | 2.64% | 5.56 | 43.19% |
| 6 | Construction & Infrastructure | 14.27% | 0.30% | 13.59% | 1.03% | 3.09% | 2.13 | 41.02% |
| 7 | Electricity | 0.21% | 6.66% | 0.001% | 0.14% | 1.16% | 2.44 | 34.26% |
| 8 | Engineering/Machinery | 3.78% | 4.55% | 5.84% | 2.12% | 2.70% | 4.06 | 34.00% |
| 9 | Ferrous Metals | 6.19% | 2.02% | 5.47% | 1.78% | 1.90% | 2.46 | 49.02% |
| 10 | Gems & Jewellery | 1.31% | 0.90% | 4.39% | 2.92% | 2.94% | 2.20 | 51.75% |
| 11 | Leather & Leather Products | 0.23% | 7.55% | 1.15% | 1.69% | 1.99% | 3.14 | 57.93% |
| 12 | NBFCs | 24.13% | 0.001% | 4.00% | 0.00% | 7.72% | 2.46 | 48.89% |
| 13 | Non Ferrous Metals | 1.34% | 5.88% | 5.49% | 1.40% | 1.75% | 3.79 | 46.64% |
| 14 | Paper and Paper Products | 0.51% | 9.05% | 1.35% | 2.12% | 1.54% | 2.73 | 57.95% |
| 15 | Petroleum Products | 2.37% | 0.40% | 11.78% | 0.01% | 0.39% | 3.72 | 32.40% |
| 16 | Processed or Packaged Foods | 2.13% | 5.02% | 1.84% | 7.00% | 8.89% | 2.27 | 64.02% |
| 17 | Rubber & Rubber Products | 0.31% | 10.25% | 1.54% | 5.64% | 6.66% | 2.22 | 72.09% |
| 18 | Sugar | 1.53% | 1.58% | 0.17% | 0.58% | 1.16% | 1.67 | 62.34% |
| 19 | Tea | 0.56% | 21.03% | 0.00% | 1.00% | 4.43% | 2.76 | 59.08% |
| 20 | Telecom | 4.87% | 0.002% | 6.23% | 0.004% | 0.77% | 4.05 | 50.32% |
| 21 | Textiles | 2.98% | 8.50% | 10.51% | 3.61% | 4.63% | 2.54 | 55.63% |
| 22 | Transport Services | 6.83% | 3.00% | 3.67% | 3.00% | 0.39% | 2.03 | 34.74% |
| 23 | Vegetable Oil & Products | 0.42% | 8.89% | 0.15% | 3.40% | 1.60% | 2.17 | 68.04% |
| 24 | Others | 19.25% | 4.00% | 10.73% | 1.60% | 4.55% | 2.30 | 44.41% |
| | Total | 100% | | 100% | | | | |



Indian Industry default correlation %: Indicative Portfolio-NIBM Computation

| SL# Industry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|----------------------|------|------|------|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|-------|------|------|------|------|------|-------|
| 1 Auto | 0.06 | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 Cement | 0.06 | 0.26 | | | | | | | | | | | | | | | | | | | | | | | |
| 3 Chemical | 0.00 | 0.00 | 0.00 | | | | | | | | | | | | | | | | | | | | | | |
| 4 Coal&Lignite | 0.44 | 0.53 | 0.03 | 10.82 | | | | | | | | | | | | | | | | | | | | | |
| 5 Computer | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | | | | | | | | | | | | | | | | | | | | |
| 6 Construction | 0.29 | 0.40 | 0.02 | 3.18 | 0.01 | 2.55 | | | | | | | | | | | | | | | | | | | |
| 7 Electricity | 0.16 | 0.21 | 0.01 | 2.06 | 0.00 | 1.19 | 0.93 | | | | | | | | | | | | | | | | | | |
| 8 Engineering | 0.02 | 0.03 | 0.00 | 0.19 | 0.00 | 0.15 | 0.08 | 0.01 | | | | | | | | | | | | | | | | | |
| 9 Ferrous Metal | 0.29 | 0.48 | 0.02 | 3.08 | 0.01 | 2.06 | 1.14 | 0.16 | 2.61 | | | | | | | | | | | | | | | | |
| 10 Gems&Jewellery | 0.35 | 0.43 | 0.02 | 3.40 | 0.01 | 2.52 | 1.31 | 0.20 | 2.56 | 4.54 | | | | | | | | | | | | | | | |
| 11 Leather&Pdcts | 0.14 | 0.25 | 0.01 | 1.27 | 0.01 | 1.12 | 0.47 | 0.10 | 1.01 | 1.60 | 1.67 | | | | | | | | | | | | | | |
| 12 Misc.Manuf | 0.28 | 0.42 | 0.02 | 2.81 | 0.01 | 2.16 | 1.06 | 0.16 | 2.08 | 2.65 | 1.31 | 2.47 | | | | | | | | | | | | | |
| 13 NBFCs | 0.26 | 0.37 | 0.01 | 2.68 | 0.01 | 1.92 | 1.10 | 0.16 | 2.11 | 2.50 | 1.10 | 2.05 | 2.61 | | | | | | | | | | | | |
| 14 Non-Ferrous Metal | 0.06 | 0.11 | 0.00 | 0.43 | 0.00 | 0.36 | 0.20 | 0.03 | 0.41 | 0.44 | 0.24 | 0.37 | 0.35 | 0.13 | | | | | | | | | | | |
| 15 Paper&Pdcts | 0.27 | 0.52 | 0.01 | 2.43 | 0.01 | 1.88 | 0.95 | 0.16 | 2.11 | 2.27 | 1.14 | 2.31 | 1.94 | 0.42 | 3.25 | | | | | | | | | | |
| 16 Petroleum Pdcts | 0.03 | 0.05 | 0.00 | 0.30 | 0.00 | 0.20 | 0.13 | 0.02 | 0.21 | 0.23 | 0.12 | 0.21 | 0.20 | 0.04 | 0.19 | 0.03 | | | | | | | | | |
| 17 Processed Food | 0.34 | 0.57 | 0.02 | 3.81 | 0.01 | 2.76 | 1.35 | 0.22 | 2.55 | 3.74 | 1.95 | 3.09 | 2.76 | 0.42 | 2.71 | 0.24 | 8.65 | | | | | | | | |
| 18 Rubber & Pdcts | 0.38 | 0.54 | 0.02 | 4.00 | 0.01 | 2.82 | 1.32 | 0.23 | 2.87 | 3.26 | 1.48 | 3.11 | 2.94 | 0.56 | 3.07 | 0.25 | 3.87 | 14.32 | | | | | | | |
| 19 Sugar | 0.48 | 0.80 | 0.03 | 5.15 | 0.01 | 3.69 | 2.05 | 0.24 | 3.73 | 4.26 | 1.85 | 3.64 | 3.47 | 0.63 | 3.21 | 0.37 | 4.22 | 5.04 | 13.73 | | | | | | |
| 20 Tea | 0.23 | 0.42 | 0.01 | 2.32 | 0.01 | 1.62 | 0.88 | 0.14 | 1.77 | 2.15 | 1.24 | 1.88 | 1.65 | 0.33 | 2.00 | 0.17 | 2.56 | 2.35 | 3.45 | 3.36 | | | | | |
| 21 Telecom | 0.06 | 0.09 | 0.00 | 0.56 | 0.00 | 0.40 | 0.24 | 0.04 | 0.42 | 0.50 | 0.27 | 0.44 | 0.51 | 0.08 | 0.41 | 0.05 | 0.77 | 0.65 | 0.64 | 0.37 | 0.38 | | | | |
| 22 Textiles | 0.32 | 0.52 | 0.02 | 2.94 | 0.01 | 2.24 | 1.17 | 0.19 | 2.63 | 3.17 | 1.50 | 2.51 | 2.47 | 0.48 | 2.67 | 0.22 | 3.31 | 3.57 | 4.09 | 2.23 | 0.55 | 3.61 | | | |
| 23 Trading | 0.03 | 0.05 | 0.00 | 0.20 | 0.00 | 0.19 | 0.10 | 0.02 | 0.22 | 0.28 | 0.16 | 0.23 | 0.24 | 0.05 | 0.24 | 0.02 | 0.33 | 0.33 | 0.29 | 0.20 | 0.07 | 0.28 | 0.05 | | |
| 24 Transport Serv | 0.22 | 0.24 | 0.02 | 2.97 | 0.00 | 1.50 | 0.98 | 0.10 | 1.40 | 1.60 | 0.66 | 1.40 | 1.32 | 0.23 | 1.15 | 0.17 | 1.68 | 2.01 | 2.65 | 1.02 | 0.28 | 1.43 | 0.11 | 1.92 | |
| 25 Veg Oil & Pdcts | 0.39 | 0.48 | 0.02 | 4.18 | 0.01 | 2.68 | 1.47 | 0.18 | 2.46 | 2.62 | 1.29 | 2.61 | 2.40 | 0.43 | 2.05 | 0.24 | 3.55 | 3.48 | 5.13 | 2.21 | 0.49 | 2.74 | 0.23 | 1.74 | 11.98 |

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Default Correlation between Farm Sector

| | Crops | Dairy | Swine | OtherLvst | Landlord | GenFarms | RuralRes | Others |
|-----------|-------|-------|-------|-----------|----------|----------|----------|--------|
| Crops | 1.0 | 0.7 | 0.7 | 0.9 | 0.4 | 0.0 | 0.0 | 0.0 |
| Dairy | 0.7 | 1.0 | 0.3 | 0.8 | 0.0 | 0.0 | 0.0 | 0.0 |
| Swine | 0.7 | 0.3 | 1.0 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 |
| OtherLvst | 0.9 | 0.8 | 0.7 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Landlord | 0.4 | 0.0 | 0.0 | 0.0 | 1.0 | 0.6 | 0.0 | 0.4 |
| GenFarms | 0.0 | 0.0 | 0.0 | 0.0 | 0.6 | 1.0 | 0.4 | 0.9 |
| RuralRes | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4 | 1.0 | 0.6 |
| Others | 0.0 | 0.0 | 0.0 | 0.0 | 0.4 | 0.9 | 0.6 | 1.0 |

NIBM Source: AgStar



Estimation of Default Correlation from Historical Rating Migrations

- The correlation of default probability between two assets, i and j , can be derived by using the following expression:

$$\rho_{i,j}^{D,D} = \frac{JDP_{i,j} - PD_i PD_j}{\sqrt{PD_i(1-PD_i)PD_j(1-PD_j)}} \quad (1)$$

- The joint default probability between two industries, say i & j ($JDP_{i,j}$), is the probability that loans in the both industries will default at the same time.
- Clearly, the correlation will be positive if the JDP is larger than the product of the univariate probabilities.
- The main difficulty is to estimate the JDP
- The joint default probability can be computed by first counting the number of possible pairs (or called joint default frequencies) migrating to default grade in every year over these 17 years. In counting the pairs, we assume simple random sampling without replacement. This process we have repeated to obtain JDP and default correlation for Investment Grades and Non Investment Grades as well as for more granular risk grades : AAA, AA, A, BBB, BB, B and CCC.

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Estimating Default Correlation: Rating Migration Approach

- We compare the pairs of defaulting firms with the total number of pairs of firms to estimate the joint default probability in different years.
- One year joint default probability of two obligors from the same rating grade (say BB)

$$\frac{(T_{i,D}^t)^2}{(N_i^t)^2}$$

- We have changed the pair-wise combination to avoid spurious negative correlation
- One year joint default probability of two obligors from different rating classes (say AA and BB)

$$\frac{T_{i,D} T_{j,D}}{N_i N_j}$$

- $T_{i,D}$ & $T_{j,D}$ are the number of defaults in a given year from respective grades and N_i and N_j are the number of borrowers in the beginning of the year in each grade.

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Estimating the JDP

- Considering that we have n years of data and not just 1, the estimator is (within same grade):

$$w_i^t = \frac{N_i^t}{\sum_{s=1}^n N_i^s}$$

$$JDP_{i,i} = \sum_{t=1}^n w_i^t \frac{(T_{i,D}^t)^2}{(N_i^t)^2}$$

- Where

Weight representing the relative Importance of a given year.

- For historical joint default migration of firms from different starting rating grades, we use the following equation:

$$JDP_{i,j} = \sum_{t=1}^n w_{ij}^t \frac{T_{i,D}^t T_{j,D}^t}{N_i^t N_j^t}$$

Where

$$w_{i,j}^t = \frac{N_i^t N_j^t}{\sum N_i^t N_j^t}$$

This method is used by Moody's KMV(URL:

http://www.moodyskmv.com/research/files/wp/Asset_Correlation_and_Portfolio_Risk.pdf.)

which is an revised version of Lucas (Journal of Fixed Income, 1995) and by Bahar and Nigam (RISK, 2001) to avoid spurious negative correlation.



Grade-wide Default Correlation: External Rating (Indian Experience)

| Default Correlation: All Industries (%) | | |
|---|--------|--------|
| | IG | NIG |
| IG | 3.58% | 12.10% |
| NIG | 12.10% | 19.30% |

- IG-IG, IG-NIG and NIG-NIG correlation estimates reveal the fact that as PD increases, default correlation also increases. Thus, correlation risk is highest in the NIG segment of the borrowers.



Default Correlations (in %)-All Countries, All Industries, 1981-2002-S&P CreditPro

| Rating | AAA | AA | A | BBB | BB | B | CCC |
|--------|-----|-------|------|------|------|------|------|
| AAA | NA | | | | | | |
| AA | NA | 0.16 | | | | | |
| A | NA | 0.02 | 0.12 | | | | |
| BBB | NA | -0.03 | 0.03 | 0.33 | | | |
| BB | NA | 0.00 | 0.19 | 0.35 | 0.94 | | |
| B | NA | 0.10 | 0.22 | 0.30 | 0.84 | 1.55 | |
| CCC | NA | 0.06 | 0.26 | 0.89 | 1.45 | 1.67 | 8.97 |

Source: Servigny & Renault (2004), Using, S&P CreditPro database of 10,000 firms and 22 years data, computed the above default correlation table.



Some Important Observations on Default Correlation (DC)

- Default correlations are generally **low** although they **decrease** as **ratings increase**
 - DCs of highly rated obligors are very small since defaults for these obligors are less and are due to obligor specific problems.
 - Lower rated obligors on the cusp of default are more susceptible to **downturns** in the economy and are, therefore, more likely to default in a group in line with shifts in the state of the general economy.
- DCs generally increase initially with time and then decrease as the **horizon** extends longer.
 - Because initial period behaviour is random and decrease over longer period incorporates the average business cycle



Important Observations on DCs..

- There is a common myth that the typical range of default correlation is between 1%-5%, but this is more an “average”, correlation may be high!
- Correlations within the same industry tend to be significantly higher than the average range shown above!
- The correlation between obligors belonging to the same industry group is **not equal to 1!**
- Instead of finding out correlation between each asset, it is better to estimate correlation across **sub-groups** (e.g. industry-specific/rating specific groupings)
 - Just imagine a portfolio consisting 100 obligors requires 5,000 pairs of default correlations! Hence it is required to group them under various sub-classes (industry, rating class etc.).

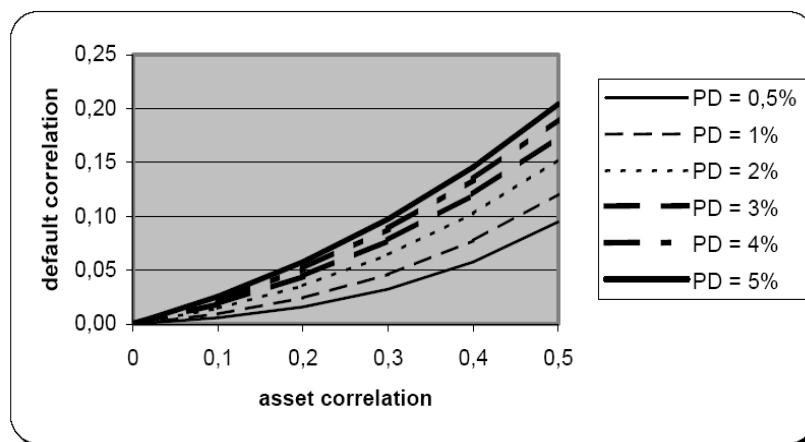


Link between Default Correlation & Asset Correlation

- **Default correlation < Asset correlation.**
- If say asset correlation is 19% between assets with PDs (0.62% and 0.25%); default correlation is only 1.68%.
- There is a common myth that the typical range of default correlation is between 1%-5%, but this is more an “average”, correlation may be high!
- Empirical evidence show, on average, asset correlation across a portfolio is in the range of 20% to 35%.
- **Default correlations** among **higher credit quality** obligors tend to be lower in comparison to lower credit quality counterparts.
- On the contrary, **asset correlations** and PDs are **inversely related** (also assumed by Basel II IRB formula).



Default Correlation as a function of Asset Correlation



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Final Output

- Economic Capital
- Risk adjusted performance measurement

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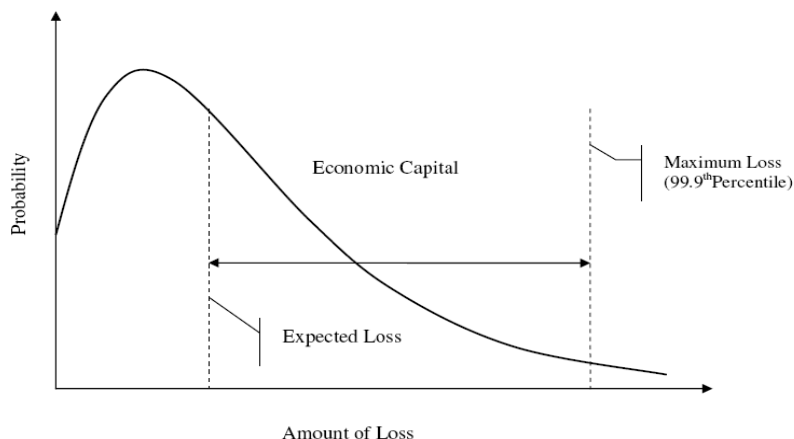
Economic Capital

- At the top of the house, economic capital gives a clear answer to the most pressing question of all:
 - Does our capital (available capital) equal or exceed the capital necessary to ensure our **survival** (economic capital) with a given level of confidence (the bank's solvency target) after taking account of our concentration risks?
- At the sub portfolio level, hard limits can be placed on notional exposure such that the economic capital as a percentage of exposure does not exceed a certain threshold (Risk criteria rather than size criteria should be the basis).

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Credit-VaR Approach



Economic capital is the tail percentile that represents the total amount of risk (value-at-risk) less the expected loss covered by the loan loss reserve

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Economic Capital

- Measured by Credit $VaR = k \cdot UL - EL$

Where k = Capital multiplier

EL = Expected Loss

UL = Unexpected Loss

From Credit VaR, we arrive at capital at risk which is also termed as “Economic Capital”.

- ❖ **Economic Capital** is the amount of capital needed to provide a cushion against the Unexpected Loss incurred in the credit portfolio.



IRB Approach

- The **internal ratings-based** approach (IRB) to credit risk is one of the most innovative elements of the New Framework because it allows banks themselves to determine certain key elements in the calculation of their capital requirements.
- The risk weights-and thus the capital charges-are determined through a combination of quantitative inputs provided either by banks or supervisory authorities, and risk weights specified by the BCBS.
- This is based on a portfolio invariant (or granular) **credit VaR approach** (similar to economic capital)
- A standardized approach to modeling credit risk across all types of banks is used for supervisory purpose for the first time.



IRB Requirements

- Meeting even the Basel II IRB requirements is **one step behind** what is required for **active portfolio management**.
- Basel II and active portfolio management require:
 - Historical data to calibrate key inputs i.e. PDs, LGDs, EADs and IT infrastructure to capture the data (at least for 5, and 7 years)
 - But there are additional though necessary costs involved:
 - Upgrading the rating systems; more granularity; two tier rating system
 - Back testing discipline: Keep the history of not only past ratings and LGDs but also of all the relevant information to reconstitute them.

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Granularity Adjustment under IRB

- Under the Asymptotic Single Risk Factor (ASRF) framework that underpins the IRB approach, it is assumed that bank portfolios are **perfectly fine-grained**, that is, that **idiosyncratic risk has been fully diversified away**, so that **economic capital depends only on systematic risk**.
- Real-world portfolios are not, of course, perfectly fine-grained. The **asymptotic assumption** might be **approximately valid** for some of the **largest bank portfolios**, but clearly would be much less satisfactory for portfolios of smaller or more specialized institutions.
- Measuring concentration risk is concern for the regulator for supervisory review process under pillar II .

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A Stylized Example of the effect of Granularity on Portfolio Risk

| Number of loans | 10 | 50 | 100 | 500 | 1,000 | 2,000 | 3,000 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| VaR(95%) | .0526 | .0508 | .0459 | .0393 | .0386 | .0378 | .0389 |
| VaR(99%) | .5263 | .1695 | .1009 | .0786 | .0773 | .0762 | .0758 |
| VaR(99.9%) | .5263 | .1864 | .1284 | .0982 | .0971 | .0950 | .0947 |

Note: Credit VaR at the specified level of confidence expressed as a fraction of total portfolio exposure. The calculations assume PD=1% and asset correlation of 20%.

- The above table illustrates how economic capital (credit VaR) varies over a sequence of seven loan portfolios with the following structure: they all contain a number of exposures to similar credits which are all of the same size with the exception of one that is ten times that size.
- As the number of credits increases in the portfolio, the importance of the single large exposure declines and the economic capital converges to the one corresponding to the infinitely granular case.



Portfolio Simulation & Economic Capital

Simulation based C-VaR Results

| Rating Target | Mapped PD% | Conf. Level | EL _P % | NPA | UL _P % | Eco-cap% | Eco-cap | Deficit over & above Reg. Capital | Eco-Cap/RWA | Tier 1% |
|---------------|------------|-------------|-------------------|---------|-------------------|----------|----------|-----------------------------------|-------------|---------|
| | | | | | | | | | | |
| | | | | Prov. % | | | | | | |
| AAA | 0.03% | 99.97% | 0.72% | 1.27% | 17.65% | 16.38% | 6755.73 | 3826.31 | 20.76% | 6.44% |
| AA | 0.10% | 99.90% | 0.72% | 1.27% | 15.58% | 14.31% | 5904.25 | 2974.83 | 18.14% | 6.44% |
| AA/A | 0.50% | 99.50% | 0.72% | 1.27% | 10.05% | 8.78% | 3622.9 | 693.48 | 11.13% | 6.44% |
| AA/A | 0.70% | 99.30% | 0.72% | 1.27% | 9.22% | 7.95% | 3277.71 | 348.29 | 10.07% | 6.44% |
| AA/A | 1% | 99% | 0.72% | 1.27% | 8.12% | 6.85% | 2825.36 | -104.06 | 8.68% | 6.44% |
| A | 1.62% | 98.38% | 0.72% | 1.27% | 6.65% | 5.38% | 2219.32 | -710.1 | 6.82% | 6.44% |
| BBB | 3.56% | 96.44% | 0.72% | 1.27% | 4.48% | 3.21% | 1322.915 | -1606.51 | 4.06% | 6.44% |

- The above result is based on Monte Carlo Simulation using historical Loss data of a bank
 - It is expected that Eco-cap/RWA would be higher than Tier 1% depending on the rating target of the bank.
 - Note that median tier 1% in AA rated US Banks is: 9.2% (source Jokivuolle and Peura, The Journal of Risk, summer 2010).
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Capital & Confidence

- How much protection is sufficient against unexpected losses?
- At Bank of America, the total amount of economic capital attributed to all of the business units is the amount that is estimated to guarantee the solvency of the bank at a **99.97% confidence level**.
- Senior management's choice of the 99.97% coverage level -- alternatively, a .03 % probability of default -- was determined by evaluating the implicit risks and default rates of public debt projected over a one-year horizon.
- 99.97% coverage level was sufficient to reduce the risk of the bank to the average levels for AA-rated companies.

Estimated Default Probabilities By Rating Class

| S&P Rating | Moody's Equivalent | Default Prob (Subsequent Year) | Distance to Default (DD) | Coverage Level |
|------------|--------------------|--------------------------------|--------------------------|----------------|
| AAA | Aaa | 0.01% | 3.72 | 99.99% |
| AA | Aa3/A1 | 0.03% | 3.43 | 99.97% |
| A | A2/A3 | 0.11% | 3.06 | 99.89% |
| BBB | Baa2 | 0.30% | 2.75 | 99.70% |
| BB | Ba1/Ba2 | 0.81% | 2.40 | 99.19% |
| B | Ba3/B1 | 2.21% | 2.01 | 97.79% |
| CCC | B2/B3 | 6% | 1.55 | 94% |
| CC | B3/Caa | 11.68% | 1.19 | 88.32% |
| C | Caa/Ca | 16.29% | 0.98 | 83.71% |



IRB-K is Designed to address UL

- If capital is set according to the gap between EL and VaR, and if EL is covered by provisions or revenues, then the likelihood that the bank will remain solvent over a one-year horizon is equal to the confidence level meaning that the needed capital per Rs. of exposure is given by:

$$C_{\alpha}(L) = q_{\alpha}(L) - E(L) = \sum_{i=1}^N w_i s_i \left[\Phi \left(\frac{\Phi^{-1}(p_i) + \sqrt{\rho_i} \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_i}} \right) - p_i \right]$$

$$E(L) = \sum_{i=1}^N w_i s_i p_i$$

- Under Basel II, capital is set to maintain a supervisory fixed confidence level (99.9%).
- Thus, the IRB capital requirements cover mainly **UL**.
- p_i is the PD, s_i is the LGD in % of exposure, $\sqrt{\rho_i}$ is the correlation between the asset return and for client i and the common factor.



Basel II IRB Approach

$$\text{Regulatory Capital} = \left(\sum_j RWA_j \right) \times 8\%$$

$$RWA = 12.5 \cdot EAD \cdot K$$

Capital requirement, K = minimum capital per unit exposure

$$K = LGD \cdot \left[N \left(\frac{N^{-1}(PD) + \sqrt{R} N^{-1}(0.999)}{\sqrt{1-R}} \right) - PD \right] \cdot MF(M, PD)$$

PD = obligor's probability of default

LGD = loss given default

R = one-factor asset correlation

MF = maturity factor

M = maturity

99.9% unexpected default losses for a one-year loan

- Asymptotically fine-grained, homogeneous portfolio

- 1-factor credit portfolio model



Asset Correlations

$$K = LGD \cdot \left[N \left(\frac{N^{-1}(PD) + \sqrt{R} N^{-1}(0.999)}{\sqrt{1-R}} \right) - PD \right]$$

- Corporate and Banking exposures

$$R = 0.12 \left(\frac{1 - e^{-50PD}}{1 - e^{-50}} \right) + 0.24 \left(1 - \frac{1 - e^{-50PD}}{1 - e^{-50}} \right)$$

- Residential mortgages

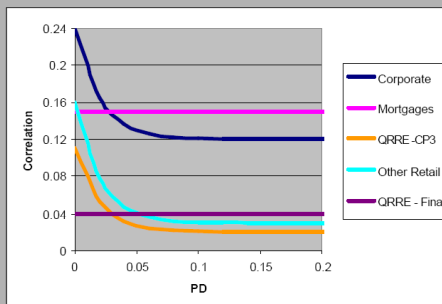
$$R = 0.15$$

- Revolving credit exposures

$$R = 0.04$$

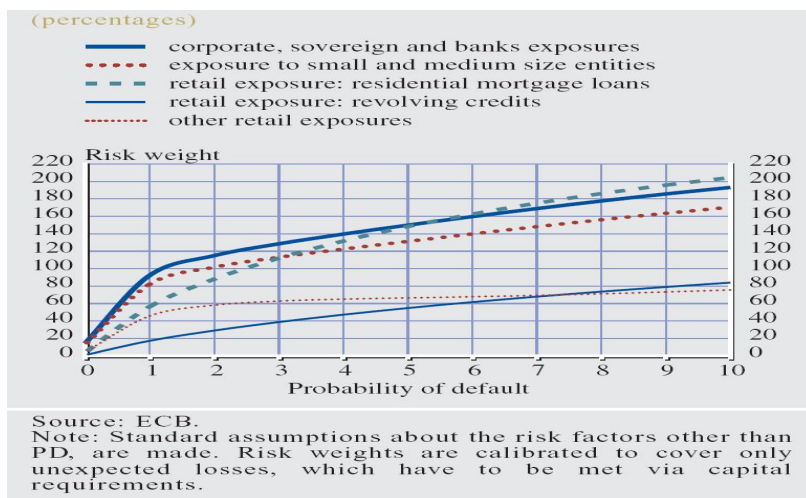
- Other retail credit exposures

$$R = 0.03 \left(\frac{1 - e^{-35PD}}{1 - e^{-35}} \right) + 0.16 \left(1 - \frac{1 - e^{-35PD}}{1 - e^{-35}} \right)$$





IRB Risk Weights across Asset Classes



IRB Risk Weight Sensitivity for UL: An Illustration

| Asset Class: | Corporate Exposure | | SME as Corporate | | Other Retail (/SME as Retail) | | Residential Mortgage | | | Qualifying Revolving Retail | |
|----------------------------|--------------------|---------|------------------|---------|----------------------------------|---------|----------------------|---------|---------|--------------------------------|---------|
| LGD: | 50% | 75% | 50% | 75% | 50% | 85% | 25% | 45% | 75% | 45% | 85% |
| Maturity: 2.5 years | | | | | | | | | | | |
| Turnover (Rs. Cr.): 50 Cr. | | | 5 Cr. | | | | | | | | |
| PD: | | | | | | | | | | | |
| 0.03% | 16.05% | 24.05% | 12.55% | 18.83% | 4.95% | 8.41% | 2.31% | 4.15% | 6.92% | 0.98% | 1.85% |
| 0.05% | 21.83% | 32.75% | 17.11% | 25.66% | 7.37% | 12.52% | 3.46% | 6.23% | 10.38% | 1.51% | 2.86% |
| 0.10% | 32.95% | 49.42% | 25.89% | 38.83% | 12.40% | 21.09% | 5.94% | 10.69% | 17.82% | 2.71% | 5.12% |
| 0.25% | 54.97% | 82.45% | 43.34% | 65.02% | 23.50% | 39.96% | 11.83% | 21.30% | 35.50% | 5.76% | 10.88% |
| 0.50% | 77.35% | 116.02% | 61.01% | 91.52% | 35.96% | 61.13% | 19.49% | 35.08% | 58.47% | 10.04% | 18.97% |
| 0.75% | 91.98% | 137.96% | 72.38% | 108.57% | 44.55% | 75.74% | 25.81% | 46.46% | 77.44% | 13.80% | 26.06% |
| 1% | 102.57% | 153.86% | 80.44% | 120.66% | 50.86% | 86.46% | 31.33% | 56.40% | 94% | 17.22% | 32.53% |
| 2% | 127.62% | 191.42% | 98.38% | 147.58% | 64.43% | 109.53% | 48.85% | 87.94% | 146.56% | 28.92% | 54.63% |
| 2.50% | 135.73% | 203.59% | 103.81% | 155.72% | 67.66% | 115.03% | 55.91% | 100.64% | 167.73% | 33.98% | 64.18% |
| 5% | 166.50% | 249.76% | 124.74% | 187.11% | 73.79% | 125.45% | 82.35% | 148.22% | 247.04% | 54.74% | 103.41% |
| 7% | 187.63% | 281.44% | 140.70% | 211.04% | 76.95% | 130.82% | 97.22% | 175% | 291.67% | 67.88% | 128.21% |
| 9% | 206.26% | 309.39% | 155.81% | 233.71% | 81.33% | 138.25% | 108.74% | 195.74% | 326.23% | 78.96% | 149.15% |
| 10% | 214.54% | 321.81% | 162.79% | 244.19% | 83.94% | 142.69% | 113.56% | 204.41% | 34.68% | 83.89% | 158.47% |
| 12% | 229.05% | 343.57% | 175.41% | 263.12% | 89.64% | 152.39% | 121.71% | 219.07% | 365.12% | 92.74% | 175.18% |
| 15% | 246.15% | 369.22% | 191.01% | 286.51% | 98.45% | 167.36% | 130.96% | 235.72% | 392.87% | 103.88% | 196.23% |
| 20% | 264.70% | 397.05% | 209.35% | 314.03% | 111.42% | 189.41% | 140.62% | 253.12% | 421.86% | 117.99% | 222.86% |



The Pro-cyclicality Issue & Basel II

- Pro-cyclicality refers to the empirical observation that banks' loan business tends to follow the same **cyclical pattern** as that of the real economy. Hence, loans typically show strong growth in an economic upturn and slow growth or even contraction in an economic downturn and so is their capital structure.
- Basel II may give rise to procyclical effects owing to the fact that the three main input parameters (PD, LGD and Correlation) especially under the IRB approach are themselves – albeit to different degrees – influenced by cyclical movements.
- To counter pro-cyclicality Effect, Basel II committee suggests that Banks should have a strong **stress testing** framework for scenario analysis.

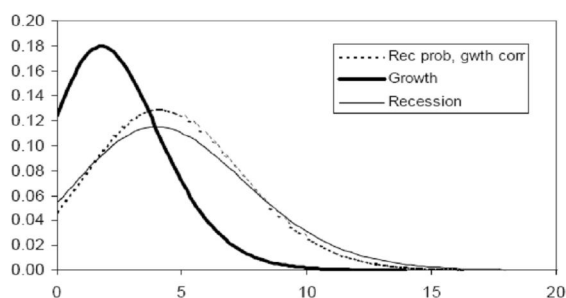


Pro-cyclicality problem

- Procyclicality involves the regulatory capital impact for expected and unexpected losses based on the rating distribution of bank portfolios.
- If IRB capital requirement becomes more dependent on the business cycle, bank's regulatory capital will increase which will limit the credit supply when the economy is slowing down and may further aggravate the condition.
- This may come because of “rating revisions” and change in “PD” or due to increase in LGD due to fall in collateral value or increase in credit exposure or due to link between PD and LGD which will ultimately impact RWA.
- The issue is that ratings and default rates respond to the cycle, the banks need to protect its capital and limit the credit supply.
- Therefore, Banks need to perform **stress test** to ensure bank's capital adequacy in times of shocks.



Credit VaR in Growth & Recession



- The growth scenario has a PD=4.31% & corr=0.77% and recession scenario has PD=8.88% & corr=1.5% in the downturn.
- The third scenario (dotted) is a hybrid case where PD=8.88% (recession) and the correlation is 0.77% (growth)

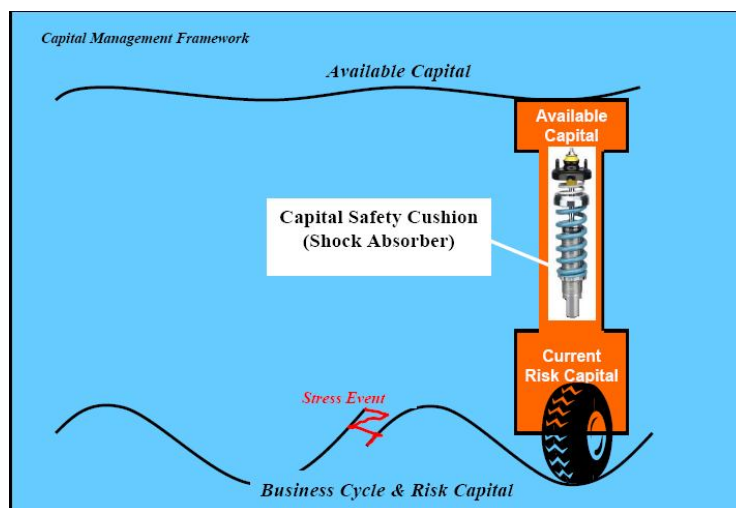
| | 95% VaR | 99% VaR | 99.7% VaR | 99.9% VaR |
|--------------------------|---------|---------|-----------|-----------|
| Growth | 6.8 | 9.0 | 10.4 | 11.8 |
| Rec prob, gwth corr | 9.6 | 11.8 | 13.4 | 14.2 |
| Recession | 10.8 | 13.6 | 15.4 | 17.2 |
| Correlation contribution | 30% | 39% | 40% | 56% |

Source: De Servigny & O Renault (2003), "Default correlation: empirical evidence", S&P working paper

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Stress Testing Bank Capital

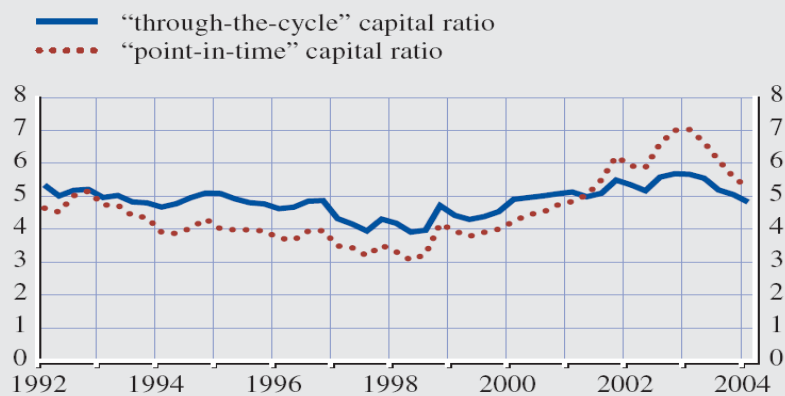


Source: Standard and Poor's



Difference in Sensitivity of Capital requirements

(percentages)



Source: Marcelo and Scheicher (2005).



Major Assumptions under Basel II IRB Approach

- Portfolio fully diversified with “infinite granularity”
 - Very large loans cause extra risk
- No uncertainty in LGD or EAD
 - Need to use “stress” or “downturn” cases
- Normal distribution of a single macro risk factor & company shocks
 - Shocks may be worse than “Normal”
- PD, LGD and EAD are estimated correctly
 - If these factors are undercounted, capital will be undercounted



RAPM based Business Strategy

- RAROC and EVA (called as RAPM framework) are credible tools and key drivers that can meet the regulatory expectations regarding the bank's conscious decision-making across business lines.
- Banks have to clearly specify the
 - Linkage between its Business Plan Formulation & Achievements with
 - Shareholders' Expectations in terms of EVA & Hurdle Rate
 - Regulator's Expectations as reflected in terms of RAROC
 - Economic Profit Vis-à-vis Profit after Tax
 - Above Business Plan has to translate into the Business Plans of its Regions and finally the Business plans of individual Branches within each region of the Bank
- The Common Thread which should drive the organization are
- RAROC & EVA

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RAROC-EVA Framework for Managing Bank Capital

- RAROC Measures performance on a risk-adjusted basis. Calculated as the economic return divided by economic capital.
- RAROC helps determine if a financial entity has the right balance between capital, returns and risk. The central concept in raroc is economic capital: the amount of capital a company should put aside needed based on the risk it runs.
- Adoption of an RAPM framework is consistent with capital management principle
- Adoption of an EC based RAPM system should be driven primarily by business demands; Pillar 2 compliance is then an added requirement
- Many supervisors 'expect' an EC approach to the ICAAP exercise

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Risk Adjusted Performance

- **RAROC** seek to more accurately measure the economic or **VaR** exposure from business activities

$$\begin{aligned} \text{RAROC} &= \frac{\text{Risk Adjusted Return}}{\text{Economic Capital}} \\ &= \frac{\text{Adjusted Net Income} - \text{Operating Expenses} - \text{EL}}{\text{VaR or Economic Capital}} \end{aligned}$$

- **Adjusted Net Income**=Total Income-Cost of Funds (Cof) × Loan Exposure
- **Expected Loss (EL)**=Obligor PD × Facility LGD × Loan Exposure
- **VaR or Economic Capital**=one year credit capital based on VaR model

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RAROC Vs. Hurdle Rate

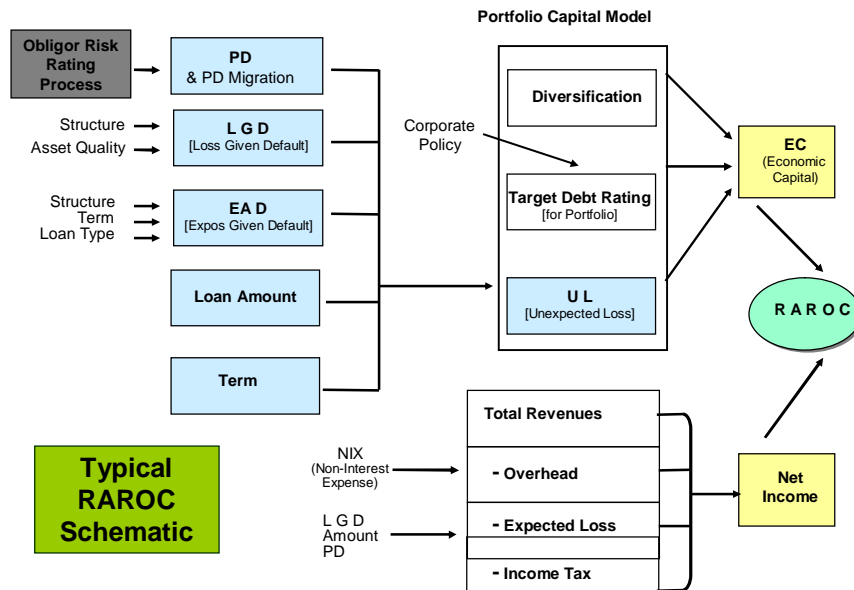
- Once calculated RAROC is to be compared with some hurdle rate reflecting the bank's cost of funds or the opportunity cost of stockholders in holding equity in the bank.
- First-generation RAROC compares RAROC to a hurdle rate; i.e., if RAROC is above a firm-wide hurdle rate (e.g., cost of equity capital), then the project is accepted. But its flaw is that it will reward risky projects: projects that have higher a RAROC but add to the firm's total risk (which is not incorporated).
- Second-generation RAROC fixes this by requiring that the project's RAROC exceed the firm's expected return; i.e., projects are accepted if $\text{RAROC} > [\text{Market return} - \text{riskless rate}] \times \text{Beta} + \text{riskless rate}$
- In RAROC models, the **hurdle rate** may be proxied by **Expected Return on Equity (ROE)** or post tax cost of equity capital (risk premium attached).
- If $\text{RAROC} > \text{Hurdle rate}$ then loan is viewed as **value adding**.

$$\text{Economic Profit} = \text{Risk Adjusted Income} - \text{HR} \times \text{Economic Capital}$$

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Case Study – Overview of Typical RAROC Model



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Uses of RAROC Tool

- Perhaps the most important use of Economic Capital and RAROC at these organizations is in guiding **strategic decisions** about growing, shrinking, or fixing businesses.
 - Accept/Reject decisions
 - Compare profitability across business segments
 - Risk Based Pricing**
 - Compensation of business units
- The RAROC calculation permits a bank to determine not only whether it is **pricing an individual loan** correctly, but whether its **overall portfolio of loans is priced correctly** or if it is carrying too much risk.
- Banks like: ABN Amro, Bank of America, Barclays, Citigroup, Deutsche Bank, ING, Union Bank of California, Bank of Montreal, Bank of Ireland, Swiss Re etc. have publicly stated that they are measuring Economic Capital and RAROC.

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A Numerical Example of RAROC based Capital Allocation

The following information is given about a bank's total business and business units, identify which business units add to share holder value?

| | | Average Yield | CoF | CoO | Expected Loss | Economic Capital |
|---------------|--------------------|---------------|------|------|---------------|------------------|
| Business Unit | Assets (Rs. Crore) | % | % | % | % | Rs. Crore |
| 1 | 25,000 | 11% | 7.0% | 2.0% | 1.8% | 438 |
| 2 | 15,000 | 12% | 7.0% | 1.0% | 2.9% | 775 |
| 3 | 10,000 | 14% | 7.0% | 0.5% | 4.1% | 2230 |
| Total | 50,000 | 11.90% | 7.0% | 1.4% | 2.6% | 3443 |

Note: Assume the cost of capital (Hurdle Rate) for the bank is 15%

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Bank's Risk Adjusted Performance Vs. Others

| Bank Name | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---------------------------------|----------|----------|----------|----------|----------|----------|
| Mid-sized PSB | | | | | | |
| Gross Advances (Rs.Cr.) | 14349.98 | 18967.99 | 24422.63 | 30422.47 | 39635.91 | 48927.12 |
| Tier I Capital% | 16.51% | 13.55% | 12.41% | 11.30% | 9.64% | 8.89% |
| CRAR% | 20.11% | 16.23% | 13.92% | 12.76% | 12.09% | 13.61% |
| ROE% | 19.62% | 13.81% | 13.82% | 14.24% | 17.38% | 18.23% |
| RAROC% | -2.41% | 5.78% | 5.65% | 10.11% | 15.84% | 19.63% |
| Hurdle Rate% | 9.52% | 9.61% | 9.90% | 10.28% | 10.41% | 9.73% |
| Large PSB1 | | | | | | |
| Gross Advances (Rs.Cr.) | 30424 | 41103 | 54644 | 63658 | 75878 | 96534.23 |
| Tier I Capital% | 6.47% | 6.07% | 7.32% | 7.79% | 7.45% | 7.41% |
| CRAR% | 12.32% | 12.09% | 11.41% | 12.80% | 12.51% | 12.01% |
| ROE% | 23.06% | 19.89% | 16.55% | 17.88% | 24.70% | 24.79% |
| RAROC% | -0.97% | 2.26% | 6.51% | 15.12% | 22.55% | 22.56% |
| Hurdle Rate% | 12.53% | 12.51% | 12.47% | 12.42% | 12.40% | 12.50% |
| Large PSB2 | | | | | | |
| Gross Advances (Rs.Cr.) | 49954 | 62726 | 76501 | 98206 | 120931 | 154703 |
| Tier I Capital% | 7.01% | 8.87% | 10.06% | 8.93% | 8.64% | 8.05% |
| CRAR% | 13.10% | 14.77% | 11.95% | 12.29% | 12.96% | 14.03% |
| ROE% | | 17.97% | 15.86% | 15.19% | 19.00% | 23.50% |
| RAROC% | 18.12% | 19.81% | 33.41% | 32.24% | 36.58% | 36.57% |
| Hurdle Rate% | 12.82% | 12.79% | 12.72% | 12.63% | 12.59% | 12.76% |
| Leading Pvt. Sector Bank | | | | | | |
| Gross Advances (Rs.Cr.) | 17744.51 | 25566.3 | 35061.26 | 46944.78 | 63426.89 | 98883.05 |
| Tier I Capital% | 8.03% | 9.60% | 8.55% | 8.57% | 10.30% | 10.18% |
| CRAR% | 11.66% | 12.16% | 11.41% | 13.08% | 13.60% | 15.09% |
| ROE% | 22.02% | 16.16% | 19.48% | 19.70% | 16.46% | 17.00% |
| RAROC% | 14.09% | 24.54% | 39.08% | 42.14% | 48.70% | 36.19% |
| Hurdle Rate% | 10.74% | 10.79% | 10.94% | 11.15% | 11.22% | 11.50% |

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Thank You
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