Shock Diffusion: Does inter-sectoral network structure matter?*

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Abstract

This paper introduces the concept of diffusion of shocks in a macroeconomic network consisting of inter-sectoral production linkages. Using sectoral and firm level data, the paper documents two empirical facts. First, sectoral output do not react contemporaneously to shocks in input sectors (it only reacts with a lag). Second, different sectors take different time horizon to respond to shocks to their input sectors. I then incorporate these features in a model of production network to study the contribution of sectoral shocks to aggregate fluctuations. I show that if sectors have different reaction horizons it leads to diffusion of shocks through the network over time which prevents the inter-sectoral linkages to form the feedback loop structure essential to generate aggregate volatility. So, the impact of a given sectoral shock lingers over a longer time period (due to diffusion) but contributes less to aggregate volatility in any given period. Finally, I use a factor model to estimate the contribution of aggregate vs idiosyncratic sectoral shocks to aggregate fluctuations in US industrial production (IP) data. I find that in the case of a diffusion adjusted network model the contribution of sectoral shocks to aggregate volatility is small and is of the same magnitude as in the case of statistical factor analysis.

KEYWORDS : Aggregate fluctuations, idiosyncratic shocks, networks, shock diffusion

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1 Introduction

It is one of the oldest debates in economics whether idiosyncratic shocks to individual sectors can generate aggregate volatility in the economy. Beginning with Lucas (1977), who argued that such shocks to individual sectors would die down in the aggregate economy due to diversification, this topic has been analyzed further in large number of papers. With the development of new tools that are available to analyze networks now, there has been a renewed interest in revisiting this old question. This debate has been carried forward in the recent paper by Acemoglu et al. (2012) who use a network argument to show that in the presence of input-output linkages, small idiosyncratic shocks can generate aggregate fluctuations depending on the structure of the production network. According to their argument, it is possible to generate such aggregate volatility from idiosyncratic shocks if the input-output network is highly asymmetric and a few big sectors provide input to a large number of other sectors.

Most of the papers with argument in favor of network hypothesis for aggregate fluctuations have a production framework where the productivity shocks propagate contemporaneously through the whole economy in just one period. Due to the static nature of the production setup the general equilibrium effects create a feedback loop in the model which allows to generate big fluctuations on the aggregate level. This raises the question- do sectors really react contemporaneously to productivity shocks in other sectors?

In the first half of the paper, I provide empirical evidence against it in two ways. First, I give macro evidence from production data at the sectoral level. It is well documented that different sectors in economy have different production horizons and there is a significant time lag between initialization and completion of any production process. For example- Humphreys et al. (2001) discuss the importance and heterogeneity of input inventories across sectors. The heterogeneity of input inventories across sectors itself speaks volume about difference in production horizon. This idea is also captured in the supply chain management and inventory literature by the concept of lead time (time taken between initiation and completion of an order). It highlights the presence of some kind of friction in the sectoral production system which takes us away from the contemporaneous production function.

But does production horizon have an impact on output adjustment? Figure 1 shows the response of durable and non-durable goods sector post Lehman bankruptcy. The non-durable goods have a lower lead time and their production can be adjusted quickly compared to the durable goods. We can clearly see that the non-durable goods sector reacted sharply to this episode and hit its lowest level in four months while the durable goods sector took much longer to cut its production (it took more than a year before the shipments and inventory level of durable goods touched their lowest level). This example documents reaction of two sectors with different production horizons to an aggregate level shock, but nonetheless shows heterogeneity in their response time.

I then provide more causal evidence using micro level data. I use firm level data and ask the question- do firms in different sectors react at different rates in response to a shock to their suppliers? To generate exogenous shocks at the supplier level, I use information on major natural disasters in the US as in Barrot and Sauvagnat (2016). These events can generate large short term impact on the sales of affected firms, which can then trickle down and affect the sales of their customers. Thus, it can be used to study propagation of shocks in downstream firms. In contrast to Barrot and Sauvagnat (2016), I estimate the impact of these shocks separately for each sector to understand if sectors differ fundamentally from each other in their response rate. I find that 1) none of the sectors react contemporaneously to the shocks and 2) there is a heterogeneity across sectors in their reaction time.

In the second half of the paper, I develop a multi-sectoral model with production linkages to study the contribution of sectoral shocks to aggregate fluctuations. Based on the empirical evidence, I add the feature of heterogeneous production horizon for different sectors in the model. I show that a model with different production horizons is sufficient to generate different diffusion rates across sectors. Now a shock to a given sector i in time period t

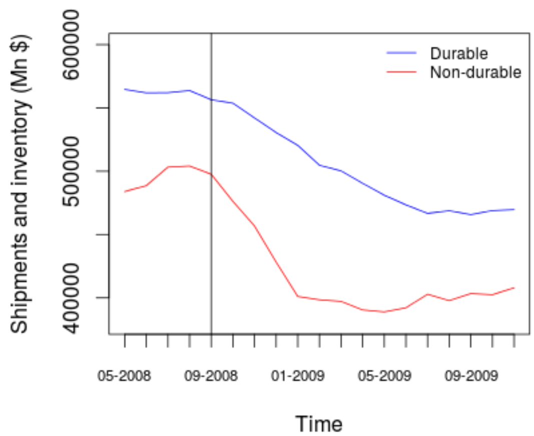

Figure 1: Reaction of durable and non-durable sectors after Lehmann bankruptcy (shown by vertical line). Non-durable goods have a smaller lead time compared to durable goods and adjusted quickly.

affects its downstream sectors at different periods of time and thus the chances of sector i generating aggregate volatility goes down. However, what kills the amplification channel is not shock diffusion on its own but the heterogeneity in diffusion rate across sectors i.e. if sectors j and k react at different times for a period t shock in sector i.

In terms of diffusion, the model presented here also generalizes the model presented in Long and Plosser (1983) as a one period diffusion model and Acemoglu et al. (2012) as a zero period diffusion model. As far as I know, all the recent papers with production networks are either zero or one period diffusion models. Since both these models have equal diffusion rate for all sectors, they are more likely to generate aggregate fluctuations from sectoral shocks.

Finally, I use the diffusion adjusted model to decompose the contribution of sectoral vs aggregate shocks in the aggregate fluctuations in US industrial production (IP) data (similar exercise as in Foerster et al. (2011)). I find that sectoral shocks now contribute only 27% to the aggregate fluctuations unlike other recent papers with production linkages, which document a much larger contribution. Accounting for unequal diffusion rates across sectors decreases the contribution of these shocks to aggregate volatility.

The paper is related to both theoretical and empirical papers in the production networks literature and its application to understanding the granular origins of aggregate fluctuations. In the case of Gabaix (2013) and Carvalho and Gabaix (2013), aggregate fluctuations are caused by large firms which contribute disproportionately both to aggregate output and aggregate fluctuations. On the other hand, Long and Plosser (1983), Bak et al. (1993), Horvath (1998) and (2000), Conley and Dupor (2003), Acemoglu et al. (2012) and Atalay (2013), generate aggregate fluctuations through sectoral inter-linkages. The empirical evidence on contribution of network linkages to generate aggregate fluctuations is provided in Di Giovanni and Levchenko (2010), Foerster et al. (2011) and Barrot and Sauvagnat (2016).

While the above mentioned papers have studied propagation of shocks through the production network, they have not focused on sectoral heterogeneity in terms of diffusion of shocks. The sectoral heterogeneity in these papers is either on how big the sector is (if network structure is taken as given) or the position of a given sector in the production network. In contrast, this paper shows that different sectors can be heterogeneous in their reaction time to shocks as well (a slightly different diffusion concept is prevalent in the network literature on study of diffusion of technology in a population, see Jackson (2005)). I provide empirical evidence for this heterogeneity in diffusion rates across sectors and show how to embed it in a network model. I find that inclusion of diffusion can significantly change the results on amplification potential of sectoral shocks in generating aggregate volatility.

The rest of the paper is organized as follows. In section 2, I present the empirical evidence in favor of heterogeneity of response rate of sectors to shocks in upstream sectors. Section 3 and 4 present models on production network and show how to introduce the concept of diffusion rate and connect it with other papers in the literature. The models are used to highlight the diversification impact of unequal diffusion over time on aggregate volatility. The diffusion adjusted model is finally taken to the data in section 5 to evaluate the contribution of sectoral shocks to aggregate fluctuations. Finally section 6 concludes.

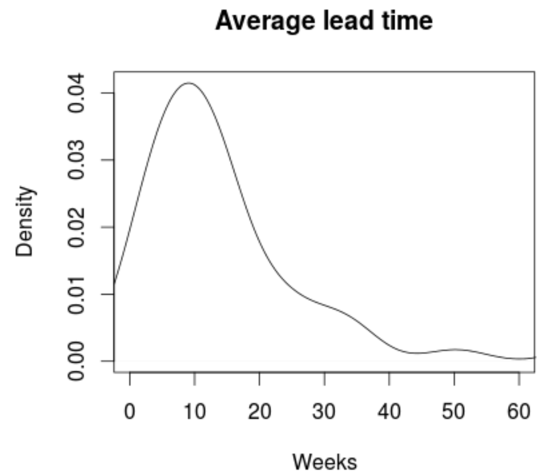

Figure 2: Average lead time across sectors (3-digit NAICS)

2 Diffusion in the data

2.1 The Macro Evidence

This idea of heterogeneous production horizon is captured in the supply chain management and inventory literature by the concept of lead time. The lead time for a given sector is the duration between conception and completion of a production process. The data counterpart of lead time is the ratio of unfulfilled shipments to value of shipments every month (source: M3 database of US census). For eg. a ratio of 1 gives a lead time of one month, which is equivalent to saying that it takes one month to complete the production after receiving an order.

The Figure 2 shows the density plot of average lead time for different sectors at the 3-digit NAICS level (monthly average lead time for the period 1991-2008). We can clearly see from the above figure that the average production horizon for sectors is approximately ten weeks, however there are a large number of sectors which plan their production in much advance. Thus lead time provides a good evidence for non-contemporaneous production decision making across sectors. This leads to our next question- if the production decisions

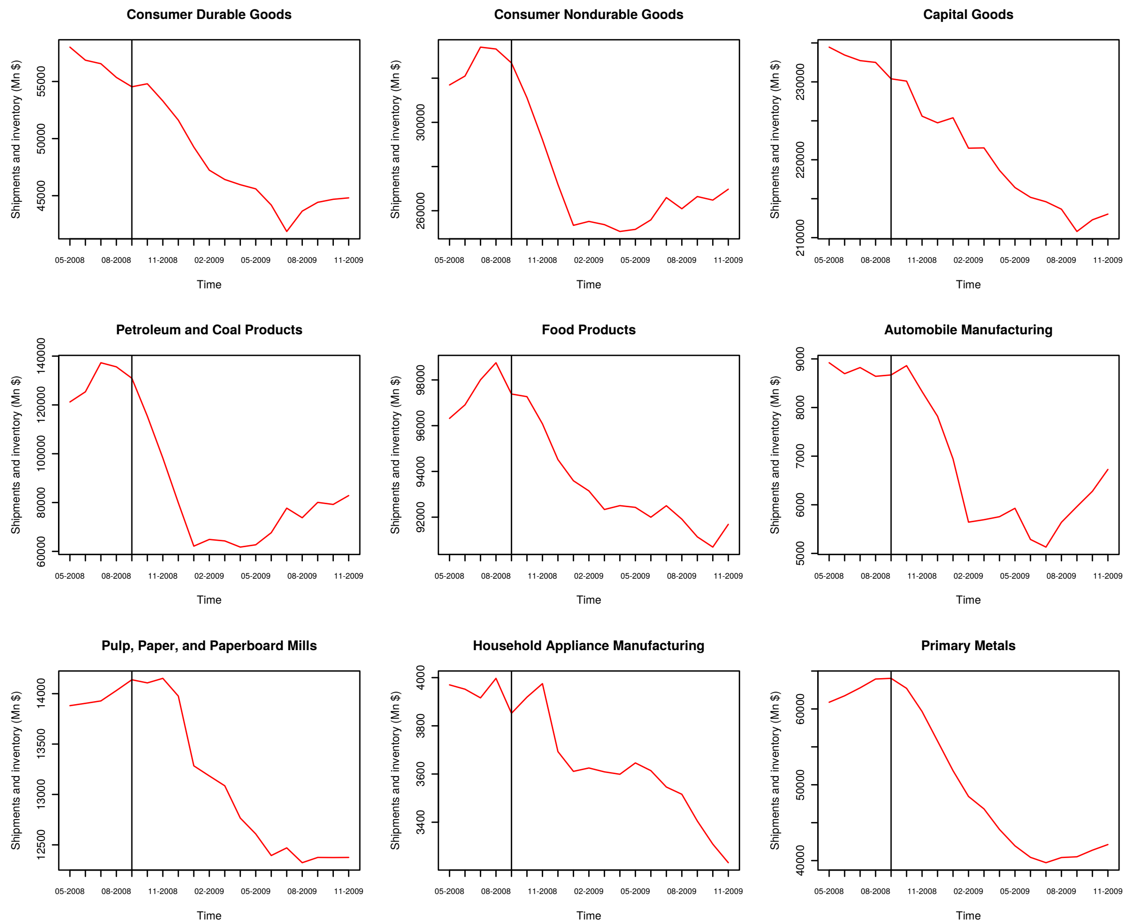

Figure 3: Reaction of various sectors after Lehmann bankruptcy

are non-contemporaneous, does it also lead to heterogeneity in the shock propagation across sectors?

To test this, the best way is to isolate sector level shocks and look at their impact on downstream sectors. However, it is difficult to isolate pure sectoral shocks which in turn makes it difficult to test their diffusion through the network. We instead use an aggregate shock, the Lehman bankruptcy (beginning of the Great Recession), and see how sectors adjusted to this common exogenous shock. Since sectors react to both aggregate and sectoral shocks, even an aggregate level shock will allow us to test for heterogeneity in reaction horizons across different sectors.

The sectoral reaction post the Lehman bankruptcy is shown in Figure 3. The figure has time period on the x-axis and value of shipments and inventory (USD million) on the y-axis for 9 different sectors [1]. We can clearly see that some sectors reacted immediately and cut down on their production thus reducing the level of shipments and inventory (for example consumer non-durables). On the other hand, sectors like consumer durables took almost an year to hit their lowest production levels. This clearly shows a heterogeneity in reaction horizon of sectors. What makes these results even more stark is the fact that the common shock in this case was known to everyone and big in magnitude, still the sectoral reaction was spread out over one year. In case of small magnitude sectoral shocks, the difference in reaction to upstream sectoral shocks can be even more pronounced, since the information on technological shock will be hard to gather.

Now we can tie together the two facts mentioned above and ask whether lead time explains the reaction horizon of sectors post the Lehman bankruptcy? In Figure 4, we give the scatterplot between lead time and the time it took to hit the lowest shipments plus inventory level after September 2008 at 3-digit NAICS sector level. Figure 4 shows that there is a positive and significant correlation between the two, which shows that on average the sectors which have higher lead time took longer to adjust post the Lehman bankruptcy announcement.

If all the sectors had reacted contemporaneously, the impact would have been much higher on the aggregate manufacturing output right after September 2008. However, the difference in production horizon (and thus the reaction horizon) across sectors cushioned the impact on the aggregate manufacturing output. It is however important to mention here that most sectors did react immediately after the aggregate shock, but only a few managed to completely adjust within a few months, while others took much longer. Thus, it is necessary to use more micro level shock to uncover diffusion channel.

[1] After a negative shock, sectors should run down current inventories for current shipments. The failure to run down inventories implies failure to cut down production. Hence it is best to use shipments plus inventory level to capture the reaction horizon of sectors.

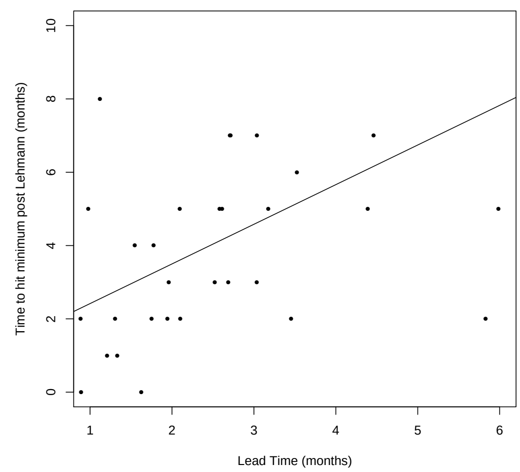

Figure 4: Scatterplot between lead time and time it took to reach minimum Shipments plus inventory after September 2008. Each dot represents a sector at 3-digit NAICS level.

2.2 The Micro Evidence

The macro evidence provided above shows that sectors indeed reacted differently after the Lehmann bankruptcy. This evidence in favor of shock diffusion in a network economy is however based on an aggregate level shock which could have impacted the sectors in more than one way (and not just through input-output linkages). Thus there is a further need to strength this evidence using micro level shocks.

In this section, I build on the work of Barrot and Sauvagnat (2016) to document the heterogeneity in response time at sectoral level after idiosyncratic shocks hit the input suppliers. The idea is to use natural disasters as an exogenous shock at the firm level. Using this exogenous shock, one can then look at the output response of firms downstream to the firm impacted by a natural disaster. Using this methodology, Barron and Sauvagnat (2016) show that sales of firms indeed take a hit after a natural disaster hits one of their supplier firms.

Identification Strategy: The same methodology (Barrot and Sauvagnat (2016)) can be augmented to find evidence for sectoral level heterogeneity in diffusion rate of shocks. If different sectors react at different time horizons after shock to their inputs, then it should also be reflected in the behavior of firms that compose these sectors. We use the following regression:

$$\triangle Sales_{is,t,t-4} = \alpha + \Sigma_{\tau=-5}^{\tau=0}\beta_{s\tau}HitsFirm_{is,t-\tau} + \Sigma_{\tau=-5}^{\tau=0}\gamma_{s\tau}HitsOneSupplier_{is,t-\tau}$$

$$+ \eta_{is} + \pi_{ts} + \varepsilon_{ist} \tag{2.1}$$

where $\triangle Sales_{is,t,t-4}$ is the sales growth for firm i in sector s between quarter t and $t-4$. $HitsFirm_{is,t-\tau}$ is a dummy variable which takes value 1 if firm is hit by a natural disaster in quarter $t-\tau$, while $HitsOneSupplier_{is,t-\tau}$ is a dummy which takes value 1 if one of the supplier of this firm is hit by a natural disaster in $t-\tau$. A firm is classified as hit by a natural disaster if it is located in a county affected by natural disaster. Finally, η_{is} and π_{ts} control for firm fixed effects and quarter-year fixed effects. The coefficient of interest here is $\gamma_{s\tau}$, which if negative and significant will imply that if a natural disaster hit a supplier in quarter $t-\tau$, it will impact the sales growth negatively for firm i in sector s at time t.

The specification in equation 2.1 is the same as used by Barrot and Sauvagnat (2016) with one important difference. It is estimated separately for each sector s instead of jointly for all sectors. The main goal for their paper was to show that shocks to input suppliers affect sales in downstream firms. On the other hand we are interested in finding out the heterogeneity in reaction horizon of firms in different sectors after a shock hits their suppliers. For example- if only γ_{a1} is the significant and negative coefficient for sector a and only γ_{b2} is the significant and negative coefficient for sector b in regression 2.1, it means that sector a has a smaller reaction horizon (equal to 1 quarter) than sector b (equal to 2 quarters).

For the identification to work, several assumptions are needed. First, the parallel trends assumption should hold i.e. firms' sales growth should be flat when the disaster has not hit any of its supplier. Second, the natural disaster should have an impact on a given firm only through the impact on its suppliers (disruption of inputs). The biggest worry here can arise from the presence of secondary plants of the customer firm being themselves present in a disaster hit county. We correct for it by dropping firms when customer-supplier headquarters are within a distance of 300 kms[2].

One of the concerns while estimating equation 2.1 is that if firms can substitute their inputs from other suppliers, one would not see any impact on sales growth. However, the significant results of this exercise will only prove the impact of such shocks. An important reason why we see the impact of these shocks on customers is because we use the data on publicly listed firms in US, which are some of the biggest firms in US[3]. A natural disaster hitting such firms can potentially knock out a significant portion of the aggregate supply from the market (lead time analysis from previous section also implies that it should be difficult to immediately replace any such supply disruption). This explains why equation 2.1 is able to find traction in the data. It also implies that the specification is robust for inferring sectoral diffusion rate of shocks because any shock to a supplier can be inferred as a partial sectoral shock.

Data Description: There are three primary datasets needed for this exercise- firm financial information, firm level network linkages and natural disasters. We use Compustat North America Fundamentals Quarterly database for firm level information, both financial data, headquarter information and firm level network. The sample is restricted to nonfinancial firms whose headquarters are located in the United States over the 1978–2013 period. We estimate the equation 2.1 for ten manufacturing sectors. The same dataset also provides information of firm level linkages. The regulation SFAS No. 131 requires firms to report any customer relationship accounting for more than 10% of sales. Finally, the data on natural disasters is collected from SHELDUS (Spatial Hazard and Loss Database for the United States) database maintained by the University of South Carolina. I use the same set of

[2]See Barrot and Sauvagnat (2016) for detailed discussion on these identification issues

[3]As shown in Gabaix (2011), the 100 largest firms themselves account for one-third of the aggregate fluctuations in US economy.

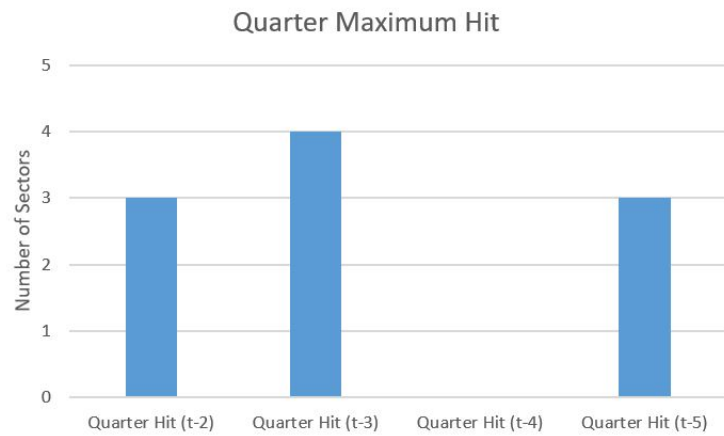

Figure 5: Distribution of reaction horizon by sectors

41 natural disasters as used by Barrot and Sauvagnat (2016) in order to keep our results comparable.

Results: The summary of results for sector level regressions based on equation 2.1 is shown in Figure 5. For each sector, we only consider the most important and significant coefficient $\gamma_{s\tau}$. If in a sector s, γ_{s2} is the most negative and significant coefficient, then in this sector firms are affected only $\tau = 2$ quarters after the shock hits their suppliers. Figure 5 reports that there are a total of three sectors which get impacted two quarters after a shock to their suppliers, four sectors which get impacted after three quarters and three sectors which get impacted after five quarters.

The results in Figure 5 immediately prove two things. First, none of the sectors react contemporaneously or even one quarter after a shock hits their suppliers. Second, there is a heterogeneity in rate of response of different sectors. Since the evidence is based on idiosyncratic shocks (exogenous due to natural disaster), it gives a causal evidence for heterogeneity in diffusion rate of shocks through sectors. In the next section, I will now present a model of input-output production economy which incorporates these findings and find the implications of sectoral shocks for aggregate economy.

3 Diffusion: Two canonical models

The phenomenon of shock diffusion can be illustrated by comparing two basic models which have been used frequently and interchangeably in the literature. The first class consists of models where shocks diffuse in the same period and affect other sectors contemporaneously. This in turn impact their own production decision in the same period and generate a feedback loop. I would call these models as zero period diffusion (0PD) models. Some of these models are presented in Carvalho (2008), Acemoglu et al. (2012), Dupor (1998) etc. The second class consists of primarily one period diffusion (1PD) model as presented in Long and Plosser (1983) where firms use inputs from the previous period for production.

In this section, I would present the basic and comparable 0PD and 1PD models as presented in Carvalho (2008) and Long and Plosser (1983). I would then use these models to highlight the difference in contribution of network interconnectivity to aggregate volatility that one can generate from considering the speed of diffusion of shocks.

3.1 0PD- Acemoglu et al. (2012)

Consider a multisector economy consisting of N different sectors indexed by $i = 1, .., N$. Each sector i produces a different good of quantity Y_{it} at date t using labor L_{it} and input X_{ijt} from other sectors $j = 1, .., N$. The Cobb-Douglas production technology used for production is given by:

$$Y_{it} = Z_{it} L_{it}^{\alpha} \prod_{j=1}^{N} X_{ijt}^{(1-\alpha)\gamma_{ij}} \tag{3.1}$$

$$\triangle Z_{it} = log(\varepsilon_{it}), \varepsilon_{it} \sim N(0, \sigma_i) \tag{3.2}$$

where Z_{it} is the productivity shock to sector i in period t. $\triangle Z_{it}$ is log-normal and i.i.d across sectors and time unless otherwise stated. X_{ijt} is the input from sector j used in the production by sector i.

The production linkages provide the source of interconnectedness between the sectors and is present in the exponent $\gamma_{ij} \geq 0$. This inter-sectoral connectivity can be completely captured by $N \times N$ matrix $\Gamma = [\gamma_{ij}]_{N \times N}$ where element ij corresponds to the share of input j for production in sector i. This matrix Γ would be referred to as input-output matrix in the rest of the paper. For now I assume that share of labor $\alpha \in (0,1)$ in production is constant across all sectors. The column sums of Γ capture the importance of a sector as an intermediate input for production in other sectors. This is defined as weighted out-degree in Acemoglu et al. (2012). I further assume that the production functions exhibit constant returns to scale which is captured by:

Assumption (A1): $\sum_{j=1}^{N} \gamma_{ij} = 1$, for all $i = 1, .., N$

On the consumption side there is a representative agent who derives utility by consuming the above mentioned N goods produced in the economy and supplies one unit of labor inelastically. The utility of this agent is given by:

$$U(C) = E_t \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{N} \theta_i ln\, C_{it} \tag{3.3}$$

$$\sum_{i=1}^{N} \theta_i = 1 \quad and \quad \theta_i > 0, \forall i \tag{3.4}$$

Since, there is no inter-temporal decision making involved in production, the above problem can be solved as a set of static problems corresponding to each time period, t. Finally, we can close the model by defining the set of resource constraints:

$$\sum_{i=1}^{N} L_{it} = 1 \tag{3.5}$$

$$C_{it} + \sum_{j=1}^{N} X_{jit} = Y_{it} \quad , \forall i = 1, .., N \tag{3.6}$$

Let $y_{it} = log\, Y_{it}$ and y_t be the vector of log sectoral output. Then, Acemoglu et al(2012) show that the competitive equilibrium of the above economy can be given by:

$$y_t = \mu_0 + [I - (1-\alpha)\Gamma]^{-1} z_t \tag{3.7}$$

where μ_0 is a N-dimensional vector of constants depending on the model parameters. Since, we are interested in aggregate growth volatility we can look at:

$$\triangle y_t = [I - (1-\alpha)\Gamma]^{-1} \varepsilon_t \tag{3.8}$$

Using the fact that all eigenvalues of $(1-\alpha)\Gamma$ are strictly less than one, we can express the above equation as a power series:

$$\triangle y_t = \left[\sum_{k=0}^{\infty} [(1-\alpha)\Gamma]^k \right] \varepsilon_t \approx [I + (1-\alpha)\Gamma]\, \varepsilon_t \tag{3.9}$$

I have ignored the second order interconnections in the above equation because it would make it easier to compare it with one period diffusion model. Although it is well documented that in a network economy second order interconnections can also matter. As I will show later, the ignored second order terms would be present in case of 1PD model as well, so we do not loose much in terms of comparison. Using the above equation, Acemoglu et al(2012) later show how aggregate volatility of economy would depend on weighted out-degree of sectors. This captures the relative importance of a sector as input to all other sectors. Given a fat-tailed distribution of weighted out-degrees one will obtain that aggregate volatility does not decay at rate $\sqrt{n}$. For now, lets look at the aggregate volatility from a practical point of view:

$$Var_{0PD}(\triangle y_t) = \Sigma_{\varepsilon\varepsilon} + (1-\alpha)^2 \Gamma \Sigma_{\varepsilon\varepsilon} \Gamma' + (1-\alpha)\Sigma_{\varepsilon\varepsilon}\Gamma' + (1-\alpha)\Gamma\Sigma_{\varepsilon\varepsilon}$$

Since we are interested in aggregate volatility, we can use an aggregate statistic:

$$Vol_{0PD}(\triangle y) = \frac{1}{N^2}\mathbf{1}' Var_{0PD}(\triangle y_t)\mathbf{1} \tag{3.10}$$

This aggregate volatility statistic is based on giving equal weight to all sectors, but it is possible to use a more realistic weighted measure when taking the model to the data. For volatility analysis, this statistic has been used frequently in the literature (see Horvath, 1998, or Dupor, 1999 or Carvalho, 2008). But comparison of the 0PD and 1PD model would be the same even if we were to consider any other sectoral weights.

3.2 1PD- Long and Plosser (1983)

The 0PD model is very similar to the classic Long and Plosser (1983) model. Now, the production in sector i in period t depends on the inputs purchased in period $t-1$. The production is given by:

$$Y_{it} = Z_{it} L_{it-1}^{\alpha} \prod_{j=1}^{N} X_{ijt-1}^{(1-\alpha)\gamma_{ij}} \tag{3.11}$$

The problem of the representative household remains the same as in the previous 0PD model. The resource constraint also remains the same except that the input X_{ijt} from sector j to i is used for production in period $t+1$:

$$C_{it} + \sum_{j=1}^{N} X_{jit} = Y_{it} \quad , \forall i = 1, .., N \tag{3.12}$$

We can again denote the log sectoral output as y_t and solve for planner's problem. Long and Plosser (1983) show that the solution to planner's problem is given by:

$$y_t = \mu_1 + (1-\alpha)\Gamma y_{t-1} + z_t \tag{3.13}$$

where μ_1 is a N-dimensional vector of constants depending on the model parameters. Since, we are interested in aggregate volatility we can work with demeaned output:

$$\triangle y_t = \left[I - (1-\alpha)\Gamma \boldsymbol{L}\right]^{-1} \varepsilon_t \tag{3.14}$$

where $\boldsymbol{L}$ is the lag operator. We can again express the above equation as a power series:

$$\triangle y_t = \left[\sum_{k=0}^{\infty} \left[(1-\alpha)\Gamma \boldsymbol{L}\right]^k\right] \varepsilon_t \approx \left[I + (1-\alpha)\Gamma \boldsymbol{L}\right] \varepsilon_t = \varepsilon_t + (1-\alpha)\Gamma \varepsilon_{t-1} \tag{3.15}$$

Similar to 0PD model, now we can write sectoral and aggregate volatility terms for 1PD diffusion model:

$$Var_{1PD}(\triangle y_t) = \Sigma_{\varepsilon\varepsilon} + (1-\alpha)^2 \Gamma \Sigma_{\varepsilon\varepsilon} \Gamma'$$

$$Vol_{1PD}(\triangle y) = \frac{1}{N^2} \mathbf{1}' Var_{1PD}(\triangle y_t) \mathbf{1} \tag{3.16}$$

One key point to differentiate 1PD model from 0PD is the timing for usage of inputs. In 0PD model, the shock from sector i immediately propagates to other sector and then affects sector i production through general equilibrium effect. This generates a feedback loop and amplification of shocks. In 1PD model on the other hand, shocks do affect other sectors but only with a lag of one period due to the time constraint on production. Now a shock to a sector i has a contemporaneous effect on itself but only a lagged one on all others, therefore there is no feedback from the other sectors to the sector i and in turn again on other sectors. This partially closes down the amplification channel as present in 0PD model.

It is a common practice to treat all these models interchangeably but as shown above they are very different in their amplification potential. This point has been ignored in other papers where the models can have extended framework involving capital and labor but inputs are produced and used in the same period. For eg. the model in Horvath (1998) solves infinite horizon problem for the social planner but still uses inputs produced in the same period. The output dependence on previous period comes only through the capital market. In terms of production linkages it is still a 0PD model and allows for contemporaneous feedback and amplification of shocks in production. On the other hand, the 1PD model uses inputs from previous periods and do not allow contemporaneous amplification of shocks through network structure.

3.3 0PD vs 1PD models

Proposition 1: The aggregate volatility in case of 0PD model is always higher than 1PD model:

$$Vol_{0PD}(\triangle y) > Vol_{1PD}(\triangle y) \tag{3.17}$$

The result here follows directly from the definition of aggregate volatility for the two models. The result will hold even if we include higher order terms in the power series expansion due to the fact that 0PD model will always include the volatility terms present in 1PD model. The reason for different aggregate volatility is due to production lag in case of 1PD model which leads to dropping out the variance term involving cross product of ε_t and $(1-\alpha)\Gamma\varepsilon_{t-1}$. Under the assumption of no auto-correlation of shocks across sectors, this cross product term is completely dropped out. But the result would hold even if there is small auto-correlation between shocks over time.

Definition : Network contribution to aggregate volatility (NC) is the fraction of volatility contributed by the terms involving network structure parameters. It can be defined as:

$$NC = 1 - \frac{\mathbf{1}'\Sigma_{\varepsilon\varepsilon}\mathbf{1}}{Vol(\triangle y)} \tag{3.18}$$

Network contribution is an important metric because it shows the importance of inter-sectoral linkages in generating aggregate volatility. If there were no intersectoral linkages, the aggregate volatility will just be the sum of sector level variances and is captured by the term $\mathbf{1}'\Sigma_{\varepsilon\varepsilon}\mathbf{1}$. The other terms in aggregate volatility contain Γ, which captures the increase in aggregate volatility due to inter-sectoral linkages.

Proposition 2: The network contribution to aggregate volatility is always higher for 0PD model:

$$NC_{0PD} > NC_{1PD} \tag{3.19}$$

Proof: The result follows directly from proposition 1. Since, the non-network term, $\mathbf{1}'\Sigma_{\varepsilon\varepsilon}\mathbf{1}$ in aggregate volatility is the same for both 0PD and 1PD models and aggregate volatility is higher for 0PD model. So we get:

$$\frac{\mathbf{1}'\Sigma_{\varepsilon\varepsilon}\mathbf{1}}{Vol_{0PD}(y)} < \frac{\mathbf{1}'\Sigma_{\varepsilon\varepsilon}\mathbf{1}}{Vol_{1PD}(y)} \tag{3.20}$$

3.4 Irrelevance of higher order diffusion process

The 1PD Long and Plosser (1983) model can be written similarly for a n-period diffusion model, with production lag of n periods. This model would seem to correspond to a slower rate of diffusion of shocks in the economy. But any such model would have no fundamental difference with 1PD model in terms of aggregate volatility. This can be summarized by:

Definition : The vector of sectoral growth rates for an n-period diffusion model will be given by:

$$\triangle y_t = [I - (1-\alpha)\Gamma \boldsymbol{L}^n]^{-1} \varepsilon_t \approx \varepsilon_t + (1-\alpha)\Gamma\varepsilon_{t-n} \tag{3.21}$$

Proposition 3: The aggregate volatility or NC do not depend on production lag i.e.:

$$Vol_{1PD}(\triangle y) = Vol_{2PD}(\triangle y) ... = Vol_{nPD}(\triangle y) \tag{3.22}$$

$$NC_{1PD}(\triangle y) = NC_{2PD}(\triangle y) ... = NC_{nPD}(\triangle y) \tag{3.23}$$

The above proposition shows that all production lags give the same value for aggregate volatility as well as the network contribution to aggregate volatility. This follows from the fact that demeaned output vector depends on two terms; current shock, ε_t and a lagged shock, ε_{t-n} times the network term $(1-\alpha)\Gamma$. In terms of diffusion process the nPD is no different than 1PD because period, t output only depends on lagged output from one other period. In case of 1PD, this input comes from period $t-1$ and in case of nPD it comes from $t-n$. So it does not have any additional dampening effects. In contrast if sectors were allowed and find it optimal to smoothen their response to shock from period $t-n$ for n periods, then the results could be different.

But at the same time, the above proposition also highlights the difference between contemporaneous production process as in 0PD model and a lagged production process in any nPD model. So for the case where firms are not allowed to smoothen their response over n periods, proposition 3 would apply and considering a production processes with more than one period lag will not change any results. For all practical purposes, one can use 0PD and 1PD models to highlight the difference caused by diffusion rate.

4 Model: Unequal diffusion rate (UDR)

Since different sectors have different production horizons, it makes sense to study a model where all sectors do not react to shocks at the same time. As discussed in the introduction and explained through figure 2, average lead time varies significantly for different sectors and determines their production horizon. The sector with small production horizon would buy its input just preceding production, while another sector with a longer production horizon might contract its inputs multiple periods before production can begin.

This difference in production horizon would create a difference in how sectors react to shocks. A sector with longer production horizon would react with a delay to the shock to its upstream sectors. Consider a sector which buys its inputs in period $t-2$ for production in period t. Since the sector is unable to tinker or change its production quickly, the shock to its supplier in period $t-2$ can affect it only in period t. In comparison, a sector which purchases its input in period $t-1$ for production in period t would react in period t if there is any shock to its suppliers in period $t-1$. In a multi-sector setting this would lead to slow diffusion of shocks through a sector with longer production horizon. Thus a multi-sector model with sectors having different production horizons would generate unequal diffusion rate of shocks in different parts of the economy.

4.1 3-sector economy

Consider a 3-sector model with the restrictions discussed above. The setting is similar to Long and Plosser (1983) with one change. Sector 1 and 2 have a small production horizon and use inputs from period $t-1$ for production in period t. On the other hand, sector 3 has a longer production horizon and uses inputs from period $t-2$ for production in period t. The production in the economy is given by:

$$Y_{it} = Z_{it} L_{it-1}^{\alpha} \prod_{j=1}^{N} X_{ijt-1}^{(1-\alpha)\gamma_{ij}} \quad \forall i = 1, 2 \tag{4.1}$$

$$Y_{3t} = Z_{3t} L_{3t-2}^{\alpha} \prod_{j=1}^{N} X_{3jt-2}^{(1-\alpha)\gamma_{ij}} \tag{4.2}$$

where Z_{it} is the productivity shock to sector i in period t and ε_t is log-normal and i.i.d. as before. The representative agent wants to maximize life-time utility and his per period utility is given by:

$$U(C_t) = \sum_{i=1}^{N} \theta_i ln\, C_{it} \tag{4.3}$$

The restrictions on the utility are same as in section 2. The resource constraint is also same, except that now sector 3 buys input in period t and uses it in period $t+2$:

$$C_{it} + \sum_{j=1}^{N} X_{jit} = Y_{it} \quad , \forall i = 1, .., N \tag{4.4}$$

Now, we can solve the planner's problem for this economy. The planner wants to maximize the expected lifetime utility of the agent subject to production functions given in (3.1) and (3.2), resource constraint (3.4) and labor market clearing conditions. This can be expressed as a value function problem:

$$V(S_t) = max \quad \{U(C_t) + \beta V(S_{t+1}|S_t) \tag{4.5}$$

where $S_t = (Y_t, Z_t)$ is the set of state variables. This problem can be solved by "guess and verify", which gives the following solution:

$$V(S_t) = k_1 ln\, Y_{1t} + k_2 ln\, Y_{2t} + k_3 ln\, Y_{3t+1} + J(Z_t) + K \tag{4.6}$$

where k_i is a set of constants given by:

$$k_i = \theta_i + \beta \sum_{j=1}^{3} k_j \gamma_{ji}, \quad \forall i = 1, 2, 3 \tag{4.7}$$

$J(Z_t)$ depends on production uncertainty parameters while K is also constant and do not depend on Y_t or Z_t. This finally gives us the consumption and input quantities at time t as given in the appendix.

Given the solution above, we can now focus on output in different sectors. It would help us compare the solution obtained here with that in the previous section. The log output for unequal diffusion rate (UDR) model is given by:

$$y_{1t} = \mu_{udr1} + (1 - \alpha) \left[\gamma_{11} y_{1t-1} + \gamma_{12} y_{t-2} + \gamma_{13} y_{t-3} \right] + z_{1t} \tag{4.8}$$

$$y_{2t} = \mu_{udr2} + (1 - \alpha) \left[\gamma_{21} y_{1t-1} + \gamma_{22} y_{t-2} + \gamma_{23} y_{t-3} \right] + z_{2t} \tag{4.9}$$

$$y_{3t} = \mu_{udr3} + (1 - \alpha) \left[\gamma_{31} y_{1t-1} + \gamma_{32} y_{t-2} + \gamma_{33} y_{t-3} \right] + z_{3t} \tag{4.10}$$

where μ_{udr} terms are constants that depend on model parameters. The above solution can be better summarized in matrix form below:

$$y_t = \mu_{udr} + (1 - \alpha) \left[\Gamma_1 y_{t-1} + \Gamma_2 y_{t-2} \right] + z_t \tag{4.11}$$

$$\triangle y_t = (1 - \alpha) \left[\Gamma_1 \triangle y_{t-1} + \Gamma_2 \triangle y_{t-2} \right] + \varepsilon_t \tag{4.12}$$

where

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad and \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \tag{4.13}$$

$$\Gamma = \Gamma_1 + \Gamma_2 \tag{4.14}$$

The above equation 4.11 captures the dynamics of the economy. The input-output matrix Γ still governs how sectoral outputs affect future production but it now gets split up in two matrices Γ_1 and Γ_2. Sectors 1 and 2 which have a production horizon of 1 period gets directly affected through Γ_1 where subscript 1 corresponds to 1-period production horizon. Sector 3, since it has a different production horizon of 2 periods gets directly impacted through Γ_2 from shocks that hit the economy in period $t-2$.

4.2 n-sector economy

Given the mechanism in the last sub-section we can easily get a reduced form solution for any n-sector economy with production linkages. Any such economy where sectors can have up to p-periods of production horizon will have a solution of VAR(P) form given by:

$$y_t = \mu_{udr} + (1-\alpha)\left[\Gamma_1 y_{t-1} + ... + \Gamma_p y_{t-p}\right] + z_t \tag{4.15}$$

$$\triangle y_t = \left[I - (1-\alpha)\left[\Gamma_1 \boldsymbol{L} + ... + \Gamma_P \mathbf{L^P}\right]\right]^{-1} \varepsilon_t \tag{4.16}$$

$$\triangle y_t \approx \left[I + (1-\alpha)\left[\Gamma_1 \mathbf{L} + ... + \Gamma_P \mathbf{L^P}\right]\right] \varepsilon_t \tag{4.17}$$

$$\Gamma = \Gamma_1 + .. + \Gamma_P \tag{4.18}$$

The solution to n-sector and P period production horizon economy has an easy reduced form as shown in equation 4.15. Since the economy now has sectors with P different production horizons, the input-output matrix Γ gets split up into P components.

4.3 1PD vs UDR models

Proposition 4: The aggregate volatility in case of 0PD and 1PD models is always higher than UDR model:

$$Vol_{0PD}(y) > Vol_{1PD}(y) > Vol_{UPR}(y) \tag{4.19}$$

Proof: It follows from the definition of $Vol_{1PD}(y)$ and $Vol_{UDR}(y)$as below:

$$Vol_{1PD}(y) = \frac{1}{N^2}\mathbf{1'}\left[\Sigma_{\varepsilon\varepsilon} + (1-\alpha)^2\Gamma\Sigma_{\varepsilon\varepsilon}\Gamma'\right]\mathbf{1}$$

$$= \frac{1}{N^2}\mathbf{1'}\left[\Sigma_{\varepsilon\varepsilon} + (1-\alpha)^2\left[\Gamma_1 + .. + \Gamma_p\right]\Sigma_{\varepsilon\varepsilon}\left[\Gamma_1 + .. + \Gamma_p\right]'\right]\mathbf{1}$$

$$> \frac{1}{N^2}\mathbf{1'}\left[\Sigma_{\varepsilon\varepsilon} + (1-\alpha)^2\left[\Gamma_1\Sigma_{\varepsilon\varepsilon}\Gamma_1' + .. + \Gamma_p\Sigma_{\varepsilon\varepsilon}\Gamma_p'\right]\right]\mathbf{1} = Vol_{UPD}(y)$$

This proposition establishes the decreases in aggregate volatility caused due to unequal diffusion rates over different sectors. The unequal diffusion rates spread the impact of a shock to sector i in period t across different periods for its different downstream consumers. It is essential for all the downstream sectors to react contemporaneously to one shock to generate substantial aggregate volatility. But unequal diffusion rates close down this amplification channel and do not allow for contemporaneous reaction for all sectors. I will further show in next sub-section below how this addition of time dimension to shock propagation can affect asymptotic properties.

The mechanism is better explained by looking at figure 6. Sector 1 is the only input supplier in the economy and supplies to all other sectors in the economy. The upper half of the figure corresponds to 1-period diffusion model. Here, a shock hits sector 1 in period t and then affects all the downstream sectors together in period $t+1$. Now compare this to the bottom half of the figure which represents an unequal diffusion rate economy where sectors 2 and 3 buy their input with 1 period production lag while 4 and 5 buy with 2 period production lag. In this second economy, the shock to sector 1 affects different parts of economy at different times. Thus on the aggregate the contribution of this shock that hits sector 1 in period t to aggregate volatility is diminished as all sectors do not react at the same time. So, even if a sector is supplier to a large number of downstream sectors its impact on aggregate volatility is diminished due to this spread of shock over time.

Proposition 5: The network contribution to aggregate volatility is also lower for UDR model:

$$NC_{0PD}(y) > NC_{1PD}(y) > NC_{UDR}(y) \tag{4.20}$$

Proof: The result follows the proof as given in Proposition 2.

Since the aggregate volatility goes down in case of UDR model, it also has a negative impact on network contribution to aggregate volatility. The diversification of the impact of period t shocks over time leads to smaller amplification of shocks due to network. This in turn decreases the contribution of network structure to aggregate volatility.

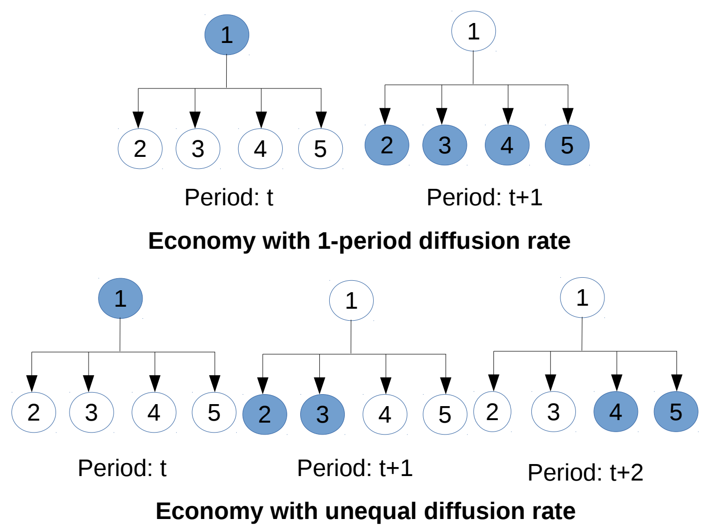

Figure 6: Shock propagation through the economy. Blue color correspond to sectors currently affected by shock that hit sector 1 in period t.

4.4 Asymptotic properties

Definition : Diffusion adjusted out-degree of a sector is the weighted out-degree measure adjusted for diffusion:

$$d_{pi} = \sum_{j=1}^{n} w_{ji}^{p} \quad where \quad w_{ji}^{p} \in \Gamma_p \tag{4.21}$$

The adjusted out-degree, d_{pi} measures the contribution of sector i as an input for period t production in other sectors which use input factors from period $t-p$. This adjusted out-degree is closely related to the weighted out-degree measure, d_i:

$$d_{pi} \leq d_i \quad \forall p, i \tag{4.22}$$

$$\sum_{p=1}^{P} d_{pi} = d_i \quad \forall i = 1, .., N \tag{4.23}$$

So, in an economy populated by sectors with P different production horizons, we would have $P \times N$ adjusted out-degree measures, d_{pi}, corresponding to lag p and sector i. The above two equations 4.22 and 4.23 follow directly from the fact that input-output matrix $\Gamma = \Gamma_1 + .. + \Gamma_P$. Since $d_{pi} \leq d_i$, it highlights the fact that sector i can be a big input supplier in the whole economy, but if sectors have different production horizons, on average the contribution of sector i production in period t an an input to other sectors can be small in subsequent periods. Thus unequal diffusion rate forces us to make the distinction between weighted out-degree, d_i and adjusted out-degree, d_{pi}.

Assumption 2(A2): The sectoral growth volatility is same across all sectors i.e. $\sigma_i = \sigma \quad \forall i = 1, .., N$.

The asymptotic results can be shown to hold for any general case where the sectoral volatility σ_i are bounded above by a finite constant. Here I have considered a simple case for illustration purpose, but can be extended as in Acemoglu et al (2012). Given assumption 3 we can now write:

Proposition 6: Under A3 and considering first order-interconnections the volatility for different diffusion models can be given by:

$$Vol_{0PD}(\triangle y)^{1/2} = Vol_{1PD}(\triangle y)^{1/2} = \Omega\left(\frac{1}{n}\sqrt{\sum_{i=1}^{n} d_i^2}\right) \tag{4.24}$$

$$Vol_{UDR}(\triangle y)^{1/2} = \Omega\left(\frac{1}{n}\sqrt{\sum_{i=1}^{n}\sum_{p=1}^{P} d_{pi}^2}\right) \tag{4.25}$$

If a few sectors provide large fraction of input supplies in the economy, this asymmetry between sectors can force the aggregate volatility to decay at a rate slower than $\sqrt{n}$. As shown in Acemoglu et al (2012), a heavy tailed distribution for d_i is enough to show that aggregate volatility decreases at a rate slower than the usual diversification argument. This result is reiterated in equation 4.24, where the zero-period output growth volatility is bounded below by average sum of squares of weighted out-degree, d_i. In contrast for an economy with unequal diffusion rates, the volatility has a different lower bound given by average sum of squares of adjusted weighted out-degree, d_{pi}.

Thus the above proposition establishes the difference in asymptotic properties that can arise depending on whether we consider shock diffusion in the economy or not. Depending on the distribution of d_i and d_{pi}, these two economies can have different decay rates for aggregate volatility. So, when we take unequal diffusion rates for different sectors into consideration it can possibly change the asymptotic properties of aggregate volatility in the economy. Also given equation 4.23, we know that the sum of d_{pi} over p periods is equal d_i. Given sufficient difference in diffusion rates across sectors, this could imply a substantial difference in distributions of d_i and d_{pi}. If d_{pi} turns out to be not so heavy tailed, then sectoral shocks would fail to generate aggregate volatility.

Another important implication of the above proposition is that input-output matrix is no longer a sufficient statistic for characterizing the role of idiosyncratic sectoral shocks in generating aggregate volatility. The aggregate volatility now depends on d_{pi} which in turn depends on both input-output structure and diffusion rate across sectors. It is possible to get the empirical counterpart of the above measure d_{pi}. The input-output matrix is usually available from national accounts, while lead time indicator can be used as a proxy for different production horizon or diffusion rate of sectors.

5 Sectoral shock decomposition

In this section, I do similar exercise as performed in Foerster, Sarte and Watson (2012) and use factor methods to decompose the industrial production (IP) into components arising from aggregate and sector specific shocks. I use structural factor analysis and see how incorporation of diffusion channel into multi-sector growth model attenuates the contribution of sector specific shocks to aggregate volatility.

5.1 Diffusion Adjusted Outdegree distribution

In this section, we do the same exercise as in Acemoglu et al. (2012) and look at the out-degree distribution in the context of US economy. The difference in this case is that we also plot the out-degrees after accounting for different diffusion rates of different sectors. The diffusion rates are proxied by lead time of different sectors. Since the different sectors in economy have different production horizons, there is a time lag between initialization and completion of production and this is captured by lead time indicator. The different lead

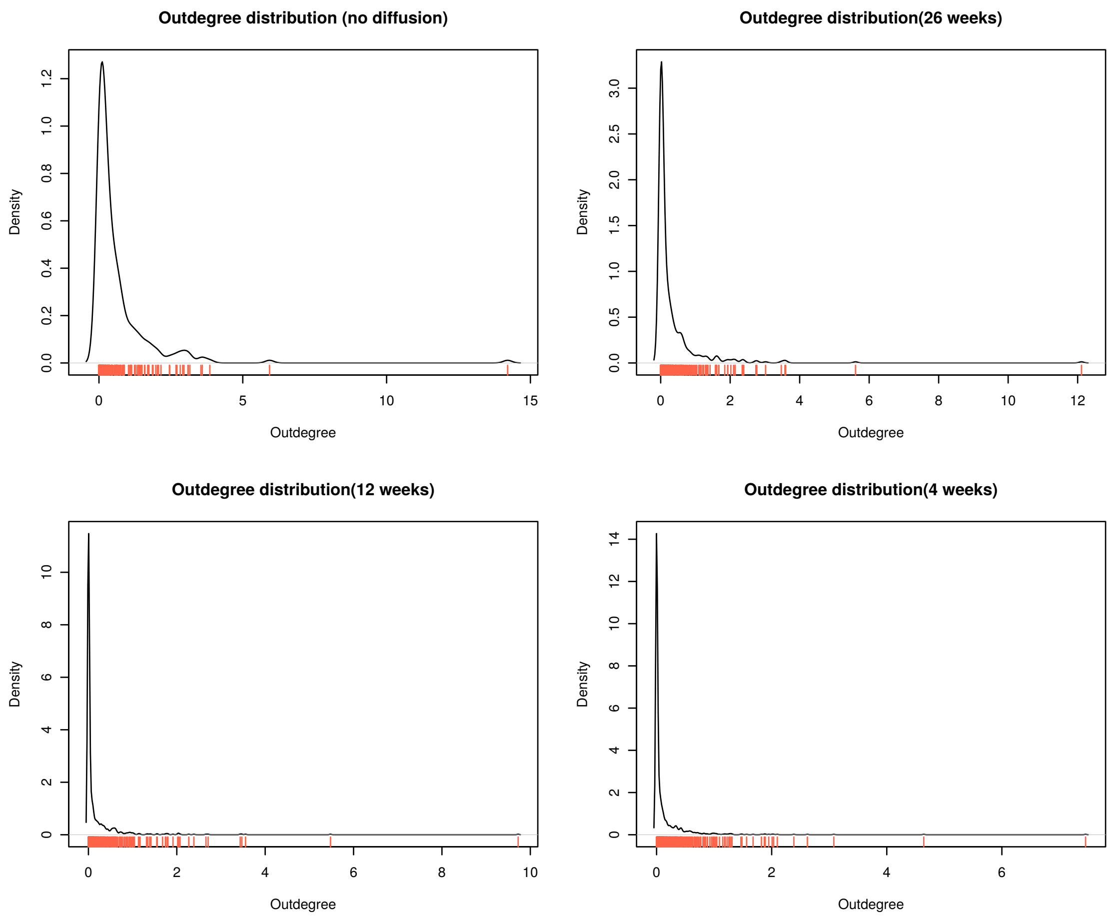

Figure 7: Distribution of diffusion adjusted out-degrees for different lead-time cutoffs

times for different sectors can be inferred from the Figure 2 in the introduction. Unlike Acemoglu et al. (2012), here I restrict my attention to the manufacturing sector of the US economy because I do not have any lead time style proxy for other sectors.

I use the detailed benchmark input–output accounts from 2007, compiled by the Bureau of Economic Analysis for the exercise in this section. BEA provides commodity-by- commodity direct requirements tables, where the typical (ij) entry captures the value of spending on commodity i per dollar of production of commodity j. As detailed above, I restrict my attention only to the manufacturing sector which gives me 237 sectors that roughly correspond to four-digit NAICS level.

As argued before, I use lead time as a proxy for diffusion rates of different sectors. The lead time of different sectors is calculated by dividing unfulfilled orders by value of shipments in a given month. I use the monthly average lead time value over the period 1991-2008 for the calculations in this section. The lead time values are not available at 4-digit level and I can only calculate it for 42 distinct sectors. These 42 sectors are both at 3 or 4-digit NAICS level. This means that lead time is not available at the same disaggregated level as input-output table which has 237 sectors. The 4-digit NAICS sectors in the direct requirements that do not have a corresponding 4-digit lead time indicator, I assign them the lead time value for 3-digit NAICS. This would give me similar diffusion rates for many sectors and would thus lead to less differentiated diffusion rates on a finer sectoral level.

Figure 7 shows the density plots of weighted out-degree for different diffusion rates depending on how we split up the economy based on sectoral lead times. The top-left panel in this figure corresponds to the case where we do not account for different diffusion rates. It is similar to the case presented in other network models like in Acemoglu et al(2012). The top right panel corresponds to dividing sectors into two categories, those with lead time less than 26 weeks and others with lead time more than 26 weeks. This gives us two different diffusion rates for the sectors in this economy where the diffusion adjusted weighted out-degree are calculated from Γ_1 and Γ_2 as in equation 4.15. The bottom left panel similarly corresponds to the case when we split sectors by lead time cutoffs 12, 24, 36 and above weeks. Finally, the bottom right panel corresponds to the case with bins created using 4, 8, 12, 24 and above week slices of lead time.

What the results in the above graphs show is that once we start accounting for differential diffusion rates, the sectors with very high weighted out-degree starts to fall. This makes it difficult to generate heavy tailed distribution of the diffusion adjusted weighted out-degree of these sectors. As compared to the top left panel where the highest outdegree was roughly 15, the bottom right panel has the highest out-degree of 8. What is more important is that the entire density shifts to the left and thus making it even less likely to generate heavy-tailed distribution.

Another important point to notice here is that these plots are generated with limited information in lead time values for many sectors. Since, the lead time data was available for only 42 sectors, a lot of sectors get assigned to the same diffusion bin corresponding to the parent NAICS level. Due to this problem a large number of sectors are present in the first bin and hence inflate the diffusion adjusted out-degrees to a certain level. But overall the diffusion mechanism decreases the likelihood of generating a heavy tailed distribution of outdegrees and thus also decreases the chances that a sectoral shock can generate aggregate fluctuations.

5.2 Overview of the IP data

I use IP data for the years 1984-2007 for the analysis in this section. The data is restricted to the above time period to keep the results comaparable to the exercise performed in Foerster, Sarte and Watson (2012). The data corresponds to 3-digit industry level NAICS classification and reported for 26 sectors. It is possible to extend the analysis and use 117 sectors i.e. 4-digit industry classification as in Foerster, Sarte and Watson (2012) instead of current 26 sectors but we are restricted by data on lead time indicator as it is reported only at 3-digit level.

The IP data is reported on a monthly frequency level but we restrict ourselves to quarterly level. The quarterly value for IP indices are constructed by taking average over the monthly values in that quarter. IP_t denotes the aggregate IP value in time period t while IP_{it} denotes the IP value for sector i in period t. We will be working with growth rates of different sectors which are denoted by g_t for the aggregate IP and as x_{it} at the sectoral level. The growth rates are then defined by $g_t = 400 \times ln\ (IP_t/IP_{t-1})$ and $x_{it} = 400 \times ln\ (IP_{it}/IP_{it-1})$.

5.3 Setup: Factor Analysis

In this section, we perform both statistical as well as structural factor analysis to decompose the aggregate fluctuations into aggregate and sectoral shocks. Let us first begin with the statistical factor analysis. Let X_t denote the vector of sectoral growth rates x_{it} in period t, then the factor model can be written as:

$$X_t = \Lambda F_t + u_t \tag{5.1}$$

where F_t is a $k \times 1$ vector of latent factors, Λ is $N \times k$ matrix of factor loadings and u_t is $N \times 1$ vector of sector specific idiosyncratic disturbances. As in classical factor analysis F_t and u_t are assumed to be mutually uncorrelated and i.i.d. with a diagonal covariance matrix for u_t. This allows us to express the covariance matrix of growth rates, X_t as $\Sigma_{XX} = \Lambda\Sigma_{FF}\Lambda' + \Sigma_{XX}$, where Σ_{FF} and Σ_{XX} are covariance matrices of F_t and u_t respectively. Since, by construction, Σ_{XX} is assumed to be diagonal, all covariance between different sectors is explained by the common factos F_t. We can use principal components to consistently estimate the factors as discussed in Stock and Watson (2000) and then use penalized least-square criterion to further select the number of factors. In the current exercise, I restrict the number of factors to two to simplify the analysis and deliver comparable results. Although the results are similar if we use just one common factor.

Now having estimated the common factors, we can use them to construct a measure for importance of aggregate shocks. We can define $R^2(F) = \bar{w}'\Lambda\Sigma_{FF}\Lambda'\bar{w}/\sigma_g^2$ as the contribution of common factors to aggregate volatility where σ_g^2 is the variance of growth rate of aggregate IP. The above formula comes from the assumption that aggregate growth rate $g_t \simeq \bar{w}'X_t$, where we have further assumed that sectoral weights $\bar{w}$, i.e. vector of contributions of sectors to overall IP, is constant over time.

The above described statistical factor analysis misses one important point that sectoral shocks can be amplified through sectoral linkages as shown in Long and Plosser (1983), Horvath (1998), Carvalho (2007) and other related papers. What this implies is that in the absence of a structural model, idiosyncratic sectoral shocks amplified through inter-sectoral linkages would appear as common shocks under statistical factor analysis. But we can use the structural models presented in the Section 3 to separate the network contribution of sectoral shocks from common shocks as done in Foerster, Sarte and Watson (2012).

We have to look at the one-period diffusion model or Long and Plosser (1983) model for carrying out structural factor analysis. The sectoral growth rate X_t is given by:

$$X_t = \left[I - (1-\alpha)\Gamma_1 \boldsymbol{L}\right]^{-1} \varepsilon_t \tag{5.2}$$

Now, sectoral innovations ε_t consist of both aggregate as well as sectoral shocks, given by:

$$\varepsilon_t = \Lambda_S S_t + \nu_t \tag{5.3}$$

where S_t is a $k \times 1$ vector of latent factors and correspond to aggregate shocks, Λ_S is $N \times k$ matrix of factor loadings while ν_t is $N \times 1$ vector of sector specific idiosyncratic disturbances. We further assume that S_t and ν_t are mutually uncorrelated and i.i.d and the idiosyncratic shocks, ν_t are uncorrelated i.e. the covariance matrix $\Sigma_{\nu\nu}$ is diagonal.

The evolution of sectoral output growth can now be expressed as a factor model:

$$X_t = \Lambda(L) F_t + u_t \tag{5.4}$$

where

$$\Lambda(L) = \left[I - (1-\alpha)\Gamma_1 \boldsymbol{L}\right]^{-1} \Lambda_S \tag{5.5}$$

and $F_t = S_t$, and

Table 1: Contribution Aggregate shocks

	Data (1)	1PD (2)	UDR (3)
R^2(S)	72%	63%	73%

$$u_t = [I - (1-\alpha)\Gamma_1 \boldsymbol{L}]^{-1} \nu_t \tag{5.6}$$

From the above equation, one can see that sectoral shocks are amplified through inter-sectoral linkages captured by the term $[I - (1-\alpha)\Gamma_1 \boldsymbol{L}]^{-1}$. Ignoring the above term is the main reason for over-estimation of contribution of aggregate shocks in aggregate volatility. To overcome this problem, one can apply factor model to ε_t, instead of X_t. The only problem is that one does not observe ε_t but it is possible to apply factor decomposition on its empirical counterpart given by:

$$\varepsilon_t = [I - (1-\alpha)\Gamma_1 \boldsymbol{L}]\, X_t \tag{5.7}$$

A similar analysis as listed above is done in Foerster, Sarte and Watson (2012). The additional exercise in this paper is is to perform a similar analysis for diffusion adjusted model. In case of diffusion adjusted model, we decompose:

$$\varepsilon_t = \left[I - (1-\alpha)\left[\Gamma_1 \boldsymbol{L} + ... + \Gamma_P \mathbf{L}^{\mathbf{P}}\right]\right] X_t \tag{5.8}$$

5.4 Results

The results of the different models discussed above are presented in table 1. The contribution of aggregate shocks is captured by the value $R^2(S)$. Column 1 corresponds to the case where we apply factor analysis to raw data. In this case, the sectoral inter-linkages do not play any role and we see that common shocks have a 72% contribution to overall volatility.

The second column in the same table corresponds to one period diffusion model or Long and Plosser (1983) model. Since this model takes into account the inter-sectoral linkages, the contribution of common shocks goes down and now only contribute 63% to the aggregate volatility. Although, the contribution of common shocks has gone down in this case but not as much as reported in Foerster, Sarte and Watson (2012). The reason being that the shocks affect downstream sectors one period later and hence attenuates some of the amplification mechanism present in their paper.

The third column needs some explanation because I have used unequal diffusion rate model in this case. I have divided the sectors into two- one with lead time less than a quarter and another with lead time more than one quarter i.e. Γ is split into Γ_1 and Γ_2. Then I applied factor method to decompose ε_t constructed using the filter $I-(1-\alpha)\left[\Gamma_1\boldsymbol{L}+\Gamma_2\mathbf{L^2}\right]$. In this case, the contribution of common shocks goes up due to the fact that sectoral shocks affect few sectors in one time period. To compensate this and achieve higher correlation between sectors, the common shocks now need to be larger to achieve the same aggregate volatility.

6 Conclusion

This paper started out to explore the idea of shock diffusion in a multi-sector economy. I provide empirical evidence on how sectors differ in the time they take to react to a given shock, which generates unequal response rate (or unequal diffusion) across sectors. Using two canonical models, I then show how a lagged production function can be used to model shock diffusion in the context of a production economy. I find that 1-period diffusion models generate less aggregate volatility when compared to 0-period diffusion models that use contemporaneous production linkages.

I then developed a more realistic diffusion model where different sectors have different production horizons and thus different diffusion rates. Under this setup, I find that introduction of shock diffusion partially closes down the important channel for shock amplification as present in the single period models with contemporaneous production linkages. Since different sectors have different shock diffusion rates, the shock to sector i at time t affects different sectors at different periods of time, thus reducing the impact of this shock on aggregate volatility in any single period. I later use this model to pin down the asymptotic properties of aggregate volatility as the number of sectors goes to infinity and again ask the question- whether idiosyncratic sectoral shocks can generate aggregate volatility in the economy after controlling for differential shock diffusion? The short answer is yes, but with a much stricter requirement. The requirement is that the diffusion adjusted weighted out-degree measure should have a heavy tailed distribution where this adjusted weighted out-degree depends on both the network structure and diffusion rates of different sectors.

In the end, the paper presents quantitative evidence to show that accounting for diffusion channel reduces the importance of inter-sectoral networks in amplifying idiosyncratic sectoral shocks. The contribution of sectoral shocks in aggregate volatility is not as high as argued in some of the recent papers. This gives important reason to further examine the diffusion channel in greater detail as it will have important implications for the direction of this literature.

References

[1] Daron Acemoglu, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012.

[2] Per Bak, Kan Chen, José Scheinkman, and Michael Woodford. Aggregate fluctuations from independent sectoral shocks: self-organized criticality in a model of production and inventory dynamics. *Ricerche economiche*, 47(1):3–30, 1993.

[3] Jean-Noël Barrot and Julien Sauvagnat. Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics*, 131(3):1543–1592, 2016.

[4] Vasco Carvalho. Aggregate fluctuations and the network structure of intersectoral trade. 2007.

[5] Timothy G Conley and Bill Dupor. A spatial analysis of sectoral complementarity. *Journal of political Economy*, 111(2):311–352, 2003.

[6] Russell Cooper and John Haltiwanger. Inventories and the propagation of sectoral shocks. *The American Economic Review*, pages 170–190, 1990.

[7] Bill Dupor. Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics*, 43(2):391–409, 1999.

[8] Andrew T Foerster, Pierre-Daniel G Sarte, and Mark W Watson. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1):1–38, 2011.

[9] Xavier Gabaix. The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772, 2011.

[10] Michael Horvath. Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics*, 1(4):781–808, 1998.

[11] Brad R Humphreys, Louis J Maccini, and Scott Schuh. Input and output inventories. *Journal of Monetary Economics*, 47(2):347–375, 2001.

[12] Matthew O Jackson. *The economics of social networks*. California Institute of Technology, 2005.

[13] Charles Jones. Input-output multipliers, general purpose technologies, and economic development.

[14] Charles I Jones. Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics*, 3(2):1–28, 2011.

[15] Christopher S Jones and Selale Tuzel. Inventory investment and the cost of capital. *Journal of Financial Economics*, 107(3):557–579, 2013.

[16] Boyan Jovanovic. Micro shocks and aggregate risk. *The Quarterly Journal of Economics*, 102(2):395–409, 1987.

[17] John B Long Jr and Charles I Plosser. Real business cycles. *Journal of political Economy*, 91(1):39–69, 1983.

[18] Robert E Lucas. Understanding business cycles. In *Essential readings in economics*, pages 306–327. Springer, 1995.