A Quantitative Theory of Installment Loans

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Abstract

We study a model of consumer debt where the only borrowing instruments are loan contracts with Equated Monthly Installments (EMIs). We calibrate the model to the contract particulars of loan-level administrative data from an Indian non-bank financial company. Indian households are infrequent borrowers and are more likely to stick to their payment schedule once they borrow. Our model accounts for most of these empirical regularities; it also finds a rationale for the ubiquitous presence of pre-payment penalties in the EMI loan market. Pre-payment penalties mitigate the adverse selection of potential defaulters into the borrower pool by reducing incentives for the pre-payers to close a loan early, thereby improving welfare.

Keywords: EMI, defaults, heterogeneity, financial frictions

JEL Codes: D14, D15, E44, G51

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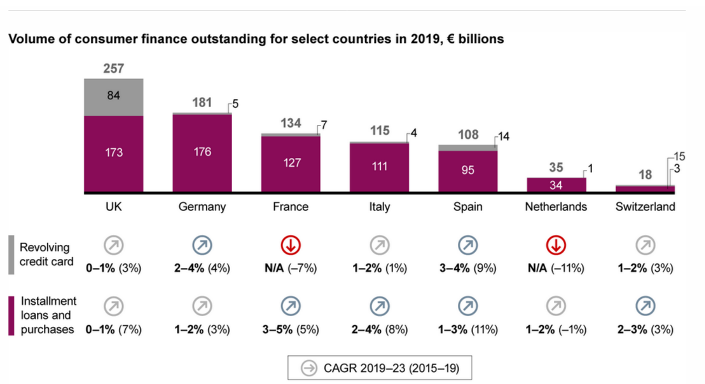
The installment plan was to consumer credit what the moving assembly line was to the automobile industry. Without it, today's trillion-dollar consumer credit industry would be inconceivable.

Lendol Calder, *Financing the American Dream*

1 Introduction

An installment loan is a consumer loan wherein the borrower commits to repay the borrowed sum, including interest, in a finite number of equal monthly payments. This form of unsecured consumer credit was common in the United States, as mentioned in the quote above. By all accounts, it is the most common form of unsecured consumer credit in most parts of the world, although official data supporting this fact is not easy to compile. The below chart, taken from a recent consulting report[1], corroborates it for a subset of European countries.

Figure 1
Importance of Installment Credit in Europe



Despite installment loans' historical and worldwide prominence, the modern quantitative theory of defaultable consumer debt has not analyzed this form of borrowing. The extant literature has modeled defaultable unsecured consumer debt either as one-period loans (Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Livshits, MacGee, and Tertilt (2007), and others) or as a revolving credit line (Raveendranathan (2020), Chatterjee and Eyigungor (2024), and others).

[1] https://www.bain.com/insights/everyone-wants-a-piece-of-consumer-finance-in-europe/. Retrieved August 13, 2024.

This paper aims to develop and apply the competitive theory of defaultable installment loans. An installment loan contract is defined by the size of the loan and the period over which it is to be repaid. Given a contract interest rate, these two characteristics determine the equal periodic payments (the so-called "installments") associated with the loan. The loan comes with the creditor's commitment not to alter the interest rates, i.e., not re-price the loan, as information is revealed over time. On the other hand, the loan comes with options on the borrower's side: The borrower can choose to pay more than the installment (and thereby end the loan before its contracted maturity date) or default.

The core of the theory of installment loans is the zero-profit pricing equation for a loan. Given the borrower's decision rules for default and prepayment and the creditors' inability to reprice the loan *ex-post*, the pricing equation generates a menu of interest rates for loans of different sizes and maturities. It encapsulates the standard one-period debt pricing equation as a particular case.

The paper also seeks to quantify the theory using data from the consumer credit market in India. Installment loans, popularly known as EMI loans—for "equal monthly installments"—are overwhelmingly the primary vehicle for extending consumer loans in India, rendering the theory's quantification particularly apposite. To do this, we relied on a widely used large-scale survey of Indian households to obtain information on the time variation in household-level incomes and the frequency of household borrowings. In addition, we obtained large samples of actual EMI loans extended by two nonbank financial companies (NBFCs) and used these to estimate the average loan-to-income ratio, as well as the default and prepayment frequencies on EMI loans. We selected parameter values to target these income, debt, default, and prepayment statistics. The quantification is successful in that parameter values can be found that deliver model statistics close to targets.

In the balance of the paper, we do three things. First, we examine the menu structure of interest rates implied by the model. We find that interest rates rise with loan size and fall with income but have a nonmonotonic relationship to maturity. Holding fixed income, interest rates increase with maturity for low debt levels and fall with maturity for high debt levels. Holding fixed debt, interest rates increase with maturity for low income and fall with maturity for high incomes. We explain this nonmonotonic relationship as the outcome of two countervailing forces.

Second, we study why households borrow in our model. Although our model is a variant of the workhorse macro model of idiosyncratic income risk with the option to save, borrow, and default, the implied behavior is unusual in some respects. We find, for instance, that neither precautionary balances nor debt can arise due to consumption smoothing in the face of income shocks. Households often save when their incomes are high, but these savings —which earn a low rate of interest— are quickly depleted when incomes are low. Consequently, the long-run precautionary balances are effectively zero. Also, borrowing to buffer income shortfalls is uncommon because the likelihood of a costly default is high, and most households prefer to lower their consumption. As a result, the model can generate debt only by occasionally shocking the marginal utility of consumption upwards: households must have a reason to spend more than their incomes. This mechanism aligns with survey respondents' reasons for taking on debt.

Finally, we study if prepayment penalties, absent in our samples of EMI loans, would be desirable. Prepayment penalties can improve welfare for the following reasons: By imposing penalties, the contract prevents borrowers who have had good income shocks from prepaying and exiting early. Since income shocks are persistent, their exit worsens the average quality of the borrower pool: the borrowers who did not get the good shocks remain, a form of *ex-post* adverse selection. Prepayment penalties mute this adverse selection and permit competitive lenders to offer lower interest rates to borrowers *ex-ante*. For our quantification, we find that a small prepayment penalty is optimal. It lowers prepayment frequency but does not drive it to zero, generating additional revenue passed back to all borrowers as lower interest rates.

The paper is organized as follows. Section 2 describes the model environment. Section 3 discusses some computational details and presents the quantification results. Sections 4 and 5 discuss how the model explains the borrowing facts and compares the model-implied menu of interest rates with the observed properties of sample contracts. Section 6 presents our findings on the desirability of prepayment penalties. Section 7 summarizes the paper and our ongoing research, and concludes.

2 Environment

A person's endowment y evolves according to a first-order Markov process $F(y'|y)$ with $y', y \in Y$. People are infinitely lived, with a discount factor of β and per-period preferences

$$u(c,\zeta)) = \frac{\zeta}{1-\gamma} c^{(1-\gamma)} \quad \gamma > 0,\ \zeta \geq 1 \tag{1}$$

where ζ is a preference shock that follows a Markov process $G(\zeta'|\zeta)$ (it is understood that $u(c,\zeta)$ is $\zeta \ln(c)$ if $\gamma = 1$).

People have access to a risk-free savings vehicle with an interest rate of $r > 0$ and to unsecured EMI consumption loans. We assume that access to the consumption loan market is limited to people in good standing, i.e., to those who do not have a record of a past default. We assume that a person can have at most one EMI loan at a time for tractability.

A person who takes out a loan of B at an interest rate of R and chooses to repay the loan over T periods is obligated to make T periodic payments of I starting the next period. The periodic payments I is given by the "EMI formula"

$$I = \frac{B \cdot R \cdot (1+R)^T}{(1+R)^T - 1} = \frac{B \cdot R}{1-(1+R)^{-T}}. \tag{2}$$

The formula indicates that for a given B and R, the EMI I depends negatively on T: I is lower for longer for maturities. Since I is what is owed each period — and can be avoided by defaulting — we expect maturity T to influence default risk via its impact on I. Therefore, in this setup, we expect the interest rate menu offered by lenders to be conditional on the persistent income level y, loan size B', and maturity T.

We assume that B belongs to a discrete set $\mathcal{B}$, maturity T belongs to a discrete set $\mathcal{T}$, the set of persistent incomes $\mathcal{Y}$, and the set of preference shocks $\mathcal{Z}$ is also discrete. Let k denote a generic tuple $(y_k, \zeta_k, B_k, T_k) \in \mathcal{K} \equiv \mathcal{Y} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{T}$. Let R_k be the interest rate for contract k. Then, (2) determines I_k as a function of R_k (and B_k and T_k). We recognize this dependence as $I_k = I(R_k)$ when needed.

2.1 Individual's Decision Problem

At any given time, the individual could be in one of three situations: she could be in a state of exclusion from credit markets following a default, which we call the exclusion status (X); she could be in good standing (i.e., not excluded from participating in the credit market) but not carrying any debt, which we call the non-indebted status (N); or she could be carrying an EMI loan, which we call the indebted status.

We now describe the Bellman equations for each of these situations.

Exclusion Status

In this situation, the person cannot have any debt and can only save. The interest rate on the asset is $r > 0$. She emerges from the exclusion state next period with some probability $\xi \in (0,1)$, and pays a cost in exclusion, denoted by $\phi(y)$.

$$v^X(A,y,\zeta) = \max_{A' \geq 0} u(c,\zeta) + \beta \mathbb{E}_{(y',\zeta')|(y,\zeta)}[\xi v^X(A',y',\zeta') + (1-\xi)v^N(A',y',\zeta')] \tag{3}$$

s.t.

$$c = y - \phi(y) + (1+r)A - A'.$$

Non-indebted Status

In this situation, the person could choose to continue in the non-indebted state, i.e., not borrow. In this case,

$$v_N^N(A,y,\zeta) = \max_{A' \geq 0} u(c,\zeta) + \beta \mathbb{E}_{(y',\zeta')|(y,\zeta)} v^N(A',y',\zeta') \tag{4}$$

s.t.

$$c = y + (1+r)A - A'$$

If she chooses to borrow, she must choose an EMI contract. Her choice set is $\mathcal{K}_y \equiv \{k \in \mathcal{K} : y_k = y\}$, i.e., the set of contracts that feature her current persistent income state. For tractability, we assume that a borrower cannot carry positive assets. So,

$$v_E^N(A,y,\zeta) = \max_{k' \in \mathcal{K}_y} u(c,\zeta') + \beta \mathbb{E}_{(y',\zeta')|(y,\zeta)} v_{k'}(B_{k'},y',\zeta') \tag{5}$$

s.t.

$$c = y + B_{k'} + (1+r)A$$

Then,

$$v^N(A,y) = \max\left\{v_N^N(A,y), v_E^N(A,y)\right\} \tag{6}$$

Indebted Status

A person who is carrying an EMI loan k has three distinct options: (i) She could continue to make the EMI payment I_k, (ii) she could pay more than the EMI payment, i.e., make a partial prepayment, or (iii) she could default.

Sticking with the EMI

$$v_k^S(B,y,\zeta) = u(c,\zeta) + \beta\mathbb{E}_{(y',\zeta')|(y,\zeta)} v_k(B',y',\zeta') \tag{7}$$
$$\text{s.t.}$$
$$c = y - \min\{I_k, (1+R_k)B\}$$
$$B' = \max\{0, B - (I_k - R_k B)\}$$

If $(1+R_k)B$ is less than I_k the loan balance inclusive of interest is less than the EMI. In this case, the "sticking" option amounts to paying off the loan balance. Note that if $(1+R_k)B < I_k$ then $B - (I_k - RB_k) < 0$ and the individual exits the period with $B' = 0$.

Partial Prepayment

Under this option, the borrower makes a payment P to reduce her principal. The set of feasible P is $\mathcal{P}_k(B) = \{P \geq 0 : B - (I_k - R_k B) - P \in \mathcal{B}\}$.

$$v_k^P(B,y,\zeta) = \max_{P \in \mathcal{P}_k(B)} u(c,\zeta) + \beta\mathbb{E}_{(y',\zeta')|(y,\zeta)} v_k(B',y',\zeta') \tag{8}$$
$$\text{s.t.}$$
$$c = y - I_k - (1+\eta)P$$
$$B' = B - (I_k - R_k B) - P$$

If $P > 0$, the borrower is required to make a prepayment penalty $\eta \geq 0$. If the borrower chooses the maximum P, she closes her loan. We denote the partial prepayment decision rule by $P_k(B,y)$ (the amount prepaid).

Default

Under this option, the individual pays a default cost of $\phi(y)$ and sheds her debt and cannot save in the period of default. Next period, she is let back into the credit market with probability $1-\xi$ or enters the exclusion state with probability ξ.

$$v^D(y,\zeta) = u(c,\zeta) + \beta\mathbb{E}_{(y',\zeta')|(y,\zeta)} \left[\xi v^X(0,y',\zeta') + (1-\xi) v^N(0,y',\zeta')\right] \tag{9}$$
$$c = y - \phi(y).$$

Then, the value of being in the indebted state with an EMI loan k is:

$$v_k(B,y,\zeta) = \max\left\{v_k^S(B,y,\zeta), v_k^P(B,y,\zeta), v^D(y,\zeta)\right\} \tag{10}$$

2.2 Competitive Pricing of an EMI Loan

Let $Q_k(B, y, \zeta)$ denote the value to the lender of contract k with balance B, current persistent state y, and shock ζ. Assuming that the lender is not paid anything in default, this value satisfies the following recursion (for compactness, we write $\mathbb{E}$ for $\mathbb{E}_{(y',\zeta')|(y,\zeta)}$):

$$\begin{aligned}
Q_k(B, y, \zeta) = {} & (1 - D_k(B, y, \zeta)) \times \\
& [\min\{I_k, (1 + R_k) \cdot B\} + (1 + r)^{-1} \mathbb{E} Q_k(\max\{0, (B - (I_k - R_k B)\}, y', \zeta')] \mathbb{1}_{\{v_k(B,y,\zeta)=(v_k^S(B,y,\zeta)\}} + \\
& [I_k + P_k(B, y, \zeta)((1 + \eta) + (1 + r)^{-1} \mathbb{E} Q_k(B - (I_k - R_k B) - P_k(B, y), y', \zeta')] \mathbb{1}_{\{v_k(B,y,\zeta)=(v_k^P(B,y,\zeta)\}}.
\end{aligned} \tag{11}$$

and the equilibrium contract interest rate R_k satisfies

$$B_k = \frac{1}{1 + r} \mathbb{E}_{(y',\zeta')|(y_k,\zeta_k)} Q_k(B_k, y', \zeta'; R_k) \text{ for all } k \in \mathcal{K}, \tag{12}$$

where we recognize that $Q_k(B, y, \zeta)$ depends directly on R_k and indirectly on R_k via $I(R_k)$ and all value functions and decision rules. Equation (12) states that at the moment of origination, the face value of the loan must equal its market value.

3 Computation and Quantification

A challenging aspect of the computation of unsecured debt models is the computation of the pricing equation for debt, in this case, equations (11) and (12). The r.h.s. of both equations are operators that take the function Q and R, respectively, as inputs and deliver "updated" Q and R functions as outputs (the l.h.s. of the two equations). Successful computation of the Q and R functions involves locating a joint fixed point of the two r.h.s. operators.

Continuity of the two r.h.s. operators is a requirement for the joint fixed point to exist, and if not satisfied, the computation might fail due to nonexistence. Since the household's choices are over a discrete set of options, the continuity of the r.h.s. operators is ensured if households choose *mixed* strategies. But our decision problem is formulated as choices over pure, not mixed, strategies. In this situation, a workaround that has become popular in the macro default literature is to attach Gumbel distributed utility shocks to payoffs that make choices probabilistic conditional on the individual's other state variables, as it would be with mixed strategies. We follow this approach.

For our application, it is sufficient to introduce payoff perturbations in two situations only. For an indebted household, the perturbations attach to the default payoff and the payoffs from each feasible payment choice. And, for a non-indebted household, it attaches to the payoff for each possible contract choice. These insertions require re-working the Bellman, Q, and R pricing equations, which are done in the appendix (to be added).

To quantify the model, we made some parametric assumptions. We assumed that deviations of log y from detrended and de-seasonalised log levels follow a common AR1 process. We estimated this AR1 process separately for households for whom we had at least 5 years of data. From these, we selected the process with the median autoregressive parameter. This gave a ρ (the autoregressive parameter) of 0.77 and a corresponding σ (standard deviation of innovations) of 0.22. We then approximated the AR1 process in levels (via Tauchen's method) by a 21-state first-order Markov chain. We assumed that $\phi(y) = \max(0, \delta_0 \cdot y + \delta_1 \cdot y^2)$. We assumed that ζ takes only two values $\{\bar{\zeta}, 1\}$, $\bar{\zeta} > 1$, and that probability of transitioning from $\bar{\zeta}$ to 1 is 1 and the probability of transitioning from 1 to $\bar{\zeta}$ is θ. Finally, we permit only 6 maturity lengths: 3, 4, 5, 6, 7, and 8 months. Our EMI loan data motivates this choice, where we observe maturities between 3 and 8 months only.

With this parametrization, the quantification involves choosing values for β, γ, $\bar{\zeta}$, θ, ξ, δ_0, δ_1, and r. We set γ to a conventional value of 2. We assumed that following a default, a person is excluded from borrowing for seven years, on average. For our monthly model, this implies $\xi = 1/84$. We assumed an annual risk-free rate of 5.3 percent, which implies $r = 0.0043$, consistent with average deposit rates in India.

For the five remaining parameters, we targeted four statistics. These are the average debt-to-income ratio in the sample of EMI loans, which mainly pins down $\bar{\zeta}$; the fraction of households in debt as measured in the large-scale sample of Indian households, which mainly pins down θ; the average default frequency, as measured in the sample of EMI loans, which mainly pins down δ_0; the average tenure of loans pinned down by δ_1; and the fraction of loans that are paid off before their maturity date as measured in the sample of EMI loans, which mainly pins down β. The estimation is done by minimizing the equally weighted sum of the squared percentage deviations of model statistics from data targets. Using this weighting scheme does not match the average tenure choice in the model to that with the data. We lower the weight on tenure and recalibrate the model. The parameter values resulting from this minimization are in the bottom panel of Table 1.

Table 1 Parameter Choices

Description	Target	Data	Model	Parameter Value
Income process	Autocorrelation	0.77	0.77	$\rho = 0.77$
Income process	Std. Deviation	0.22	0.22	$\sigma = 0.22$
IES	Conventional	-	-	$\gamma = 2$
Risk-free rate	-	0.0043	0.0043	$r = 0.0043$
Exclusion duration	Conventional	-	84	$\xi = 0.011$
Utility shock	Debt-to-Income	1.08	1.23	$\bar{\zeta}= 3.68$
Prob of U shock	Fraction in Debt	0.08	0.07	$\theta = 0.012$
Cost of default	Def Frequency	0.016	0.015	$d_0 = 0.17$
Tenure	Avg. Tenure	4.15	8	$d_1 = 0.28$
Discount factor	Preclosure Rate	0.08	0.79	$\beta = 0.75$

The identified parameters generate moments from the model that match most of those from the data. However, in the model, the borrower chooses the maximum available tenure every time.

4 Default Incentives and the Interest Rate Menu

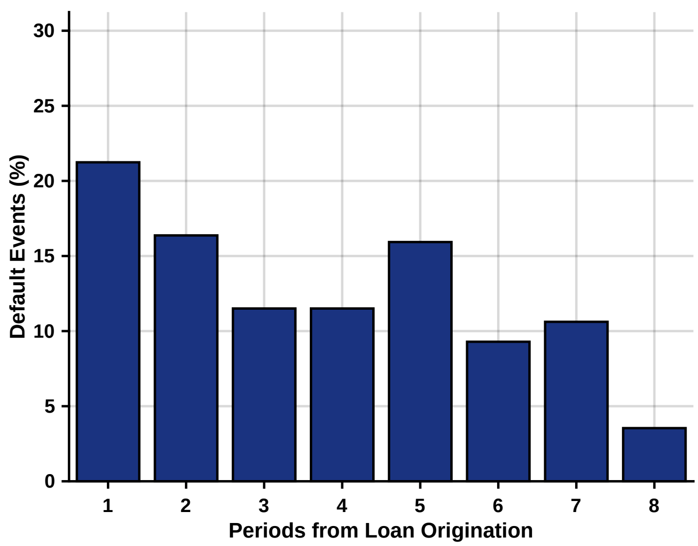
The borrowing behavior of households depends on the interest rate menu households face, which, in turn, depends on a borrower's default incentives. If default is ruled out, competition would make the interest rate on every contract equal to the risk-free rate regardless of loan size or maturity.

Default Incentives:

The value in the indebted state, $v^k(B,y,\zeta)$, decreases in B. The logic for this is as follows. Imagine a second individual with contract k and state y,ζ but with a debt level of $B-\Delta$, where Δ is a small positive number. It is feasible for this individual to take the same action each period as the individual with debt B. When the B individual pays her debt off, the $B-\Delta$ individual will also pay her debt off (because she has been following the decisions of the B individual up to that time), and she will experience a higher consumption level because she would have arrived into that period with less debt. Hence, the $B-\Delta$ individual has a feasible policy rule that, along paths that involve paying off the debt at some point, gives her more utility than the B individual. Therefore $v^k(B-\Delta,y,\zeta) > v^k(B,y,\zeta)$ for any $\Delta > 0$.

Since the payoff from default is independent of B, viz (9), the property that $v^k(B,y,\zeta)$ is decreasing in B implies that if it is optimal to default on a debt level of B, it must be optimal to default on a higher debt level. In other words, for each y, ζ and k, there is a threshold debt level $\bar{B}_k(y,\zeta)$ such that there is a default if and only if $B > \bar{B}_k(y,\zeta)$. Since the outstanding debt level declines over the lifetime of a loan, the threshold property implies that the probability of default on a loan falls over time. This is because the set of (y,ζ) states for which repayment is optimal expands over time as the level of indebtedness declines. In other words, the default hazard on an EMI loan declines as the loan gets closer to maturity. Figure 2 plots the default hazard rate. Apart from period 5 the hazard depicts that chances of default are higher closer to the loan origination date.

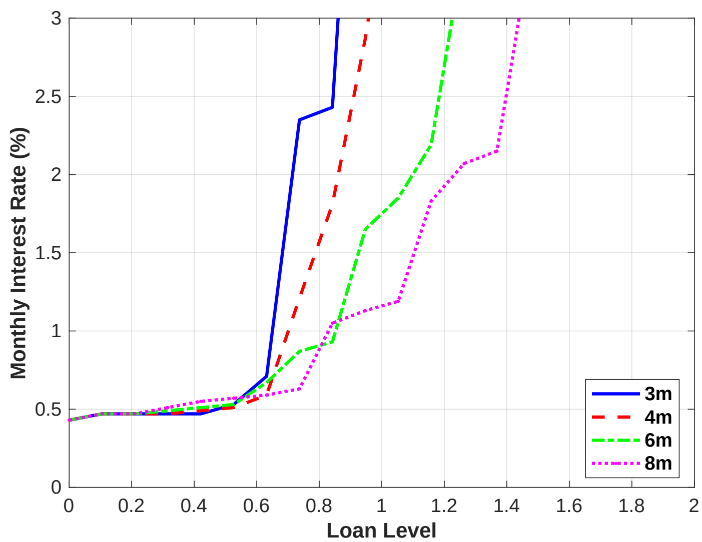
Figure 2
Default Hazard Rate



The quantity I does not appear explicitly as an argument in $v^k(B,y,\zeta)$ because it enters implicitly through the contract indicator k — which determines I_k — and does not change as the contract progresses through time. Nevertheless, we can consider a change in I_k holding fixed B, y, ζ, and R. All else the same, an increase in I_k lowers current utility since consumption is lower for all payment choices — see the budget constraint equations in (7) and (8). However, a higher I_k also means that for all payment choices, the loan will amortize *faster*, i.e., B' will be lower, meaning the continuation value will be higher. The current utility effect is a force toward increasing default probability, while the continuation value effect is a force toward decreasing default probabilities. Thus, the overall impact of a higher I_k is ambiguous.

The impact of y on default incentives is also ambiguous. An increase in y will raise $v^k(B,y,\zeta)$ since the decision rule for a lower level of y remains feasible at a higher level of y and affords strictly greater consumption. But the value in default $v^D(y,\zeta)$ also increases in y, so we cannot be sure how default incentives are affected. However, the assumption that the cost of default rises with y attenuates the increase in $v^D(y,\zeta)$ with y and ensures, in our computations, that individuals repay for high y levels and default for low y levels. In other words, there is a default threshold $\bar{y}_k(B,\zeta)$ such that default occurs if and only if $y < \bar{y}_k(B,\zeta)$.

Figure 3
Interest Rate, Loan Size and Maturity



Interest Rate Menu w.r.t. Loan Size:

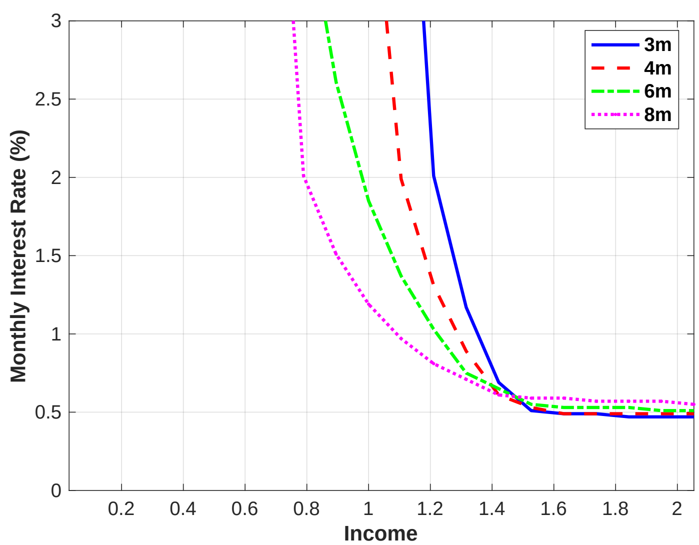
Figure 3 plots the interest rate against loan size for all the maturities available in our model. The income level for which these schedules are drawn is the median income level of 1. The curves are upward-sloping for all maturities. The reason for this is that holding fixed maturity and interest rate, a bigger loan lowers the value of indebtedness (as discussed previously) and, therefore, increases the set of (y,ζ) combinations for which default is optimal. Hence, equilibrium interest rates must rise with the level of debt. However, there is another channel at work. As interest rates rise, the EMI formula (2) implies that I_k also rises. And, as also discussed previously, this effect lowers the $\bar{B}_k(y,\zeta)$ schedules and further raises default probabilities.

Figure 3 shows that the relationship between loan maturity and interest rates depends on the size of the loan. This reflects our previous discussion about the ambiguous effect of I_k on default incentives. The figure shows that for small loan sizes, the faster amortization effect is the dominant effect and shorter maturities go with lower equilibrium interest rates. For larger loans, the I_k effect overwhelms the faster amortization effect and the longer maturity loans go with lower equilibrium interest rates.

Interest Rate Menu w.r.t. to Persistent Income Level:

Figure 4 plots the interest rate schedule for different levels of y. These schedules are drawn for a debt level of 1.05, i.e., for a debt level approximately equal to the median B. There are no interest rates for persistent income below about 0.78. This is because no contract can be offered at an interest rate less than the maximum interest rate allowed in the model (a monthly interest rate of 10 percent). As income rises, the first contract to become available is for a maturity length of 8 months, the most extended maturity available in our model, and, therefore, the lowest I_k. For higher incomes, successively lower maturities with successively higher I_k become available. We see a switch in the ordering of the graphs as well: For high levels of income, the shorter-maturity loans carry a lower interest rate than longer-maturity loans. The faster amortization effect is the dominant one in this range, while the I_k effect is the dominant one for lower income levels.

Figure 4
Interest Rate, Income and Maturity



5 Why Do Indian Households Borrow and Why Do They Default?

Figure 5
Reasons for Borrowing

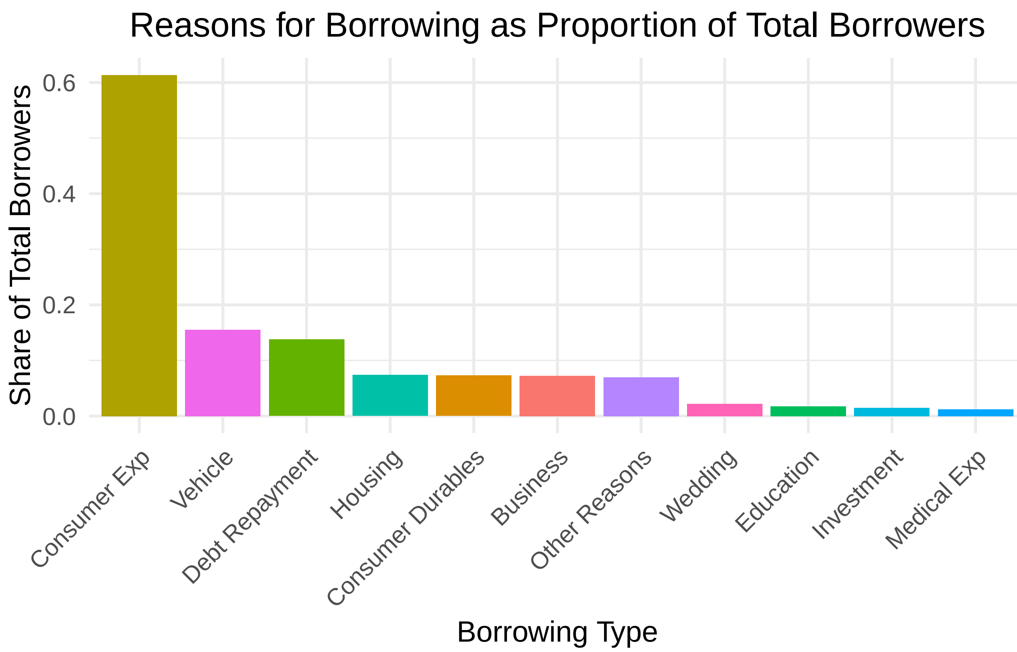


Figure 5 displays the frequencies of the respondents' different reasons for borrowing. The main reason for borrowing is consumption, with a frequency of 60 percent. Debt taken on to support consumption could be due to shortfalls in income or unexpected expenditures. On the other hand, vehicles, consumer durables, weddings, and medical expenses are best considered as expenditure shocks. These categories together account for about 27 percent. If we attribute half of the 60 percent attributed to consumption to expenditure shocks, such shocks would be the dominant reason for consumer borrowing in India.

The incidence of borrowing in our model by (y, ζ) state is shown in Table 2. Borrowing due to an income shortfall alone is uncommon: Only 8.16 percent of all borrowings in the model occur when $\zeta = 1$, and all such borrowings occur for the third lowest persistent income level. No borrowings occur for other low-income states because interest rates in low-income states are generally relatively high.

In contrast, our model always shows positive borrowing when hit by the ζ shock unless the borrower is already indebted (in which case she cannot borrow). This is because people do not accumulate any precautionary balances, so borrowing is the only way to meet the sudden expenditure demand.[2] Comparing the frequencies in the column for $\zeta > 1$ with the column for the long-run, or ergodic, distribution of y, we see that the frequency is close but always slightly less. This gap is because the high zeta shock is sometimes realized when the household is already indebted and cannot borrow.

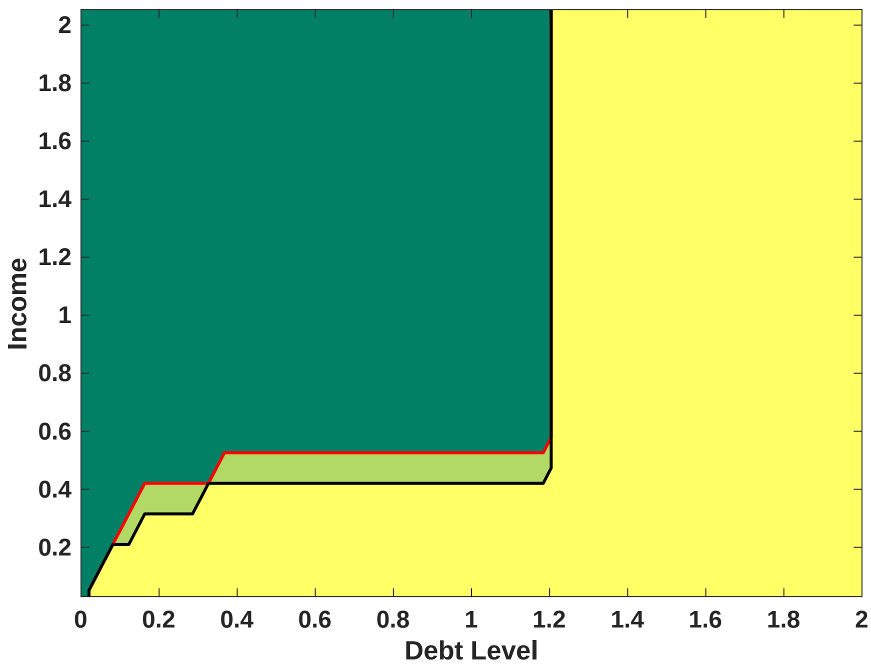
Table 2
Percentage of borrowing instances by (y, ζ) states

	$\zeta_1 = 3.68$	$\zeta_2 = 1$	**Ergodic** y
y_1	$\times$	$\times$	0.19
y_2	$\times$	$\times$	0.30
y_3	0.58	8.16	0.71
y_4	1.11	0	1.35
y_5	2.33	0	2.43
y_6	4.03	0	3.98
y_7	4.71	0	5.85
y_8	8.45	0	7.96
y_9	8.41	0	9.92
y_{10}	10.69	0	11.35
y_{11}	10.20	0	11.89
y_{12}	10.93	0	11.38
y_{13}	9.62	0	9.97
y_{14}	7.24	0	7.90
y_{15}	5.29	0	5.95
y_{16}	3.84	0	3.94
y_{17}	2.18	0	2.36
y_{18}	1.06	0	1.28
y_{19}	0.68	0	0.72
y_{20}	0.29	0	0.30
y_{21}	0.09	0	0.19

Figure 6 shows a plot of the default regions. The black line demarcates the default region (in yellow) from the repayment region (in light and dark green) when the ζ shock is 1, and the red line demarcates it when $\zeta > 1$. In either situation, the default set has the threshold property: Default occurs for income levels below the red or black line. Notice that there are no repayment states for debt levels above approximately 1.2. This is because the arbitrary contract for which the default decision rule is plotted is for a debt of 1.2. The relevant debt levels in this contract are the ones below it.

[2] Even though earnings are very volatile, people are impatient enough (and the savings interest rate low enough) that they do not have an incentive to save.

Figure 6
Default Regions



6 On the Desirability of Prepayment Penalties

We study the effect on borrower quality pool and interest rates of a prepayment penalty of 5 percentage points ($\eta = 0.05$).

Figure 7 shows that the quality of the borrower pool in terms of the expected income of surviving borrowers is higher with the prepayment penalty. This is the average income across borrowers in the simulation for an arbitrary contract through its entire tenure. Intuitively, debt being expensive, borrowers with a higher realization of y in any period after the inception of a loan are more likely to pay in full before the tenure ends. However, the prepayment penalty deters the borrower from doing so, forcing higher-income borrowers to remain longer in the pool.

Figure 8 plots the interest rate associated with median income and an arbitrary tenure for all levels of debt. The figure shows that interest rates are lower in a model with a prepayment penalty. The perfectly competitive risk-neutral borrower transfers the revenue from such a penalty by offering lower rates on the average loan.

Figure 7
Borrower Quality

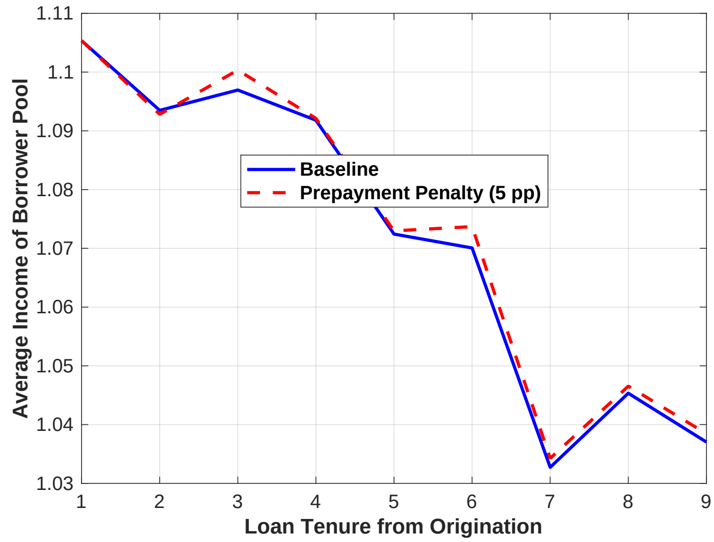
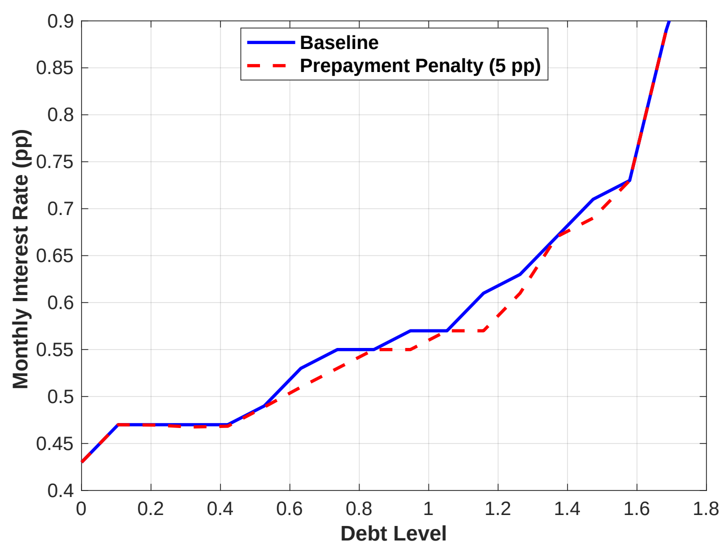
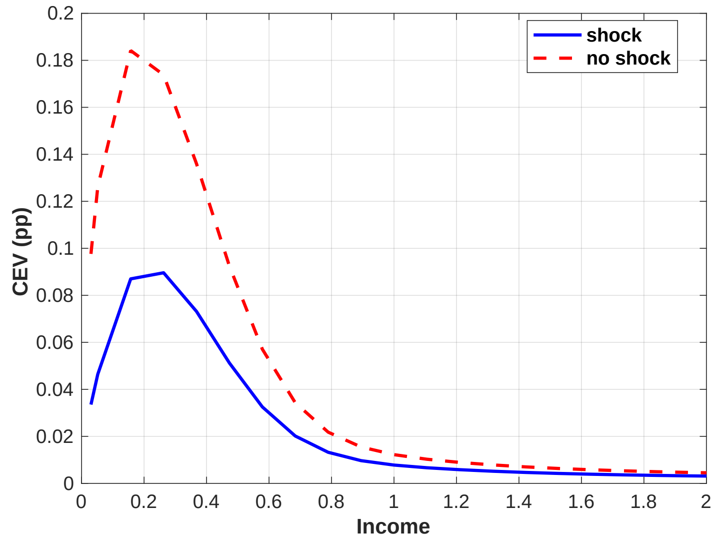


Figure 8
Interest Rates with Penalty



The effect of such penalties on welfare is ambiguous. While a small penalty creates incentives for the high-income borrower to stay in the pool, it also exposes them to default risk due to the elongation of the life of the loan. On the positive side, the revenue from the penalty reflects on the lower interest rates. If the penalty imposed is prohibitively high,

Figure 9
Welfare gains due to prepayment penalty



no borrower prepays, and no additional revenue accrues to the lender. This situation results in higher default risk and higher associated interest rates. This non-monotonicity alludes to the existence of an optimal rate of pre-payment penalty.

Figure 9 plots consumption equivalent variation (CEV): percentage of consumption required to be transferred to a borrower in the baseline model to make him/her indifferent to a world with a pre-payment penalty. The figure reports the CEV in a state with no debt and plots it against the income. The blue (solid) line depicts a high borrowing shock ($\zeta > 1$) state, and the dashed (red) line is the one for the low borrowing shock ($\zeta = 1$) state. The CEV is greater or equal to zero for all income levels. This shows that a pre-payment penalty is valuable for borrowers at all levels of income: the interest rate channel dominates the excess defaults channel. The CEV rises for low-income levels, reaches a peak, and then drops to zero. This non-monotonicity is because of limited access to credit at lower levels of income, and the higher chances of incidence of the pre-payment penalty at higher levels of income.

The ζ shock lowers the CEV for all levels of income. This is because $\zeta > 1$ induces more borrowing. A bigger loan is associated with both higher default risk and higher chances of making prepayments later. The former force reduces the benefit of lower interest rates, while the latter is a direct cost of the penalty. When $\zeta = 1$ borrowing levels are lower, if at all. This makes the world with pre-payment penalties more valuable.

7 Summary, Ongoing Research, and Conclusion

This paper builds and analyzes a model of consumer debt with EMI loans as the sole form of borrowing. The borrower chooses a contract from an offered menu. Each contract comprises an amount and a tenure, and the lender determines the interest rate. These three objects determine the installment payment due every month over the chosen tenure. This contract structure is different from the vast literature on consumer debt and defaults which mostly assumes one-period loans, and the literature on sovereign debt because here the lenders commit not to reprice the loans as information unravels for each borrower. The loan pricing equation takes this commitment into account as it generates the equilibrium interest rates.

This paper is a work in progress. Our ongoing work which we plan to include in the next version includes an attempt to match the tenure choices of the borrowers better by including loan supply restrictions that would create incentives for the lenders to extend shorter loans. Secondly, in the current setup, the borrowers are not allowed to refinance their loans. We are currently working towards introducing this option which we believe would have a significant impact on model moments. Third, we plan to compare the current contract setup to that of a one-period loan quantitatively. It would help shed light on the important aspects by which they differ.

This paper is the first of its kind that builds a model of EMI loans. With Fintech lending and buy-now-pay-later schemes becoming popular among borrowers by the day versions of this model can be used to infer properties of a plethora of such loan contracts and can be used to devise welfare-improving features into such contracts.

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